ON THE ROLE OF WATER VAPOR IN THE ENERGETICS OF THE GENERAL CIRCULATION OF THE ATMOSPHERE (*) (**)

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ABSTRACT — The role of water vapor in the energetics of the general circulation of the atmosphere is studied and discussed. The water vapor is the most important absorber of solar energy in the atmosphere, and hence, its distribution influences the form of the energy input into the system. Through the release of latent heat it generates zonal available and eddy available potential energy. The available potential energy of the disturbances is partly supplied by the non adiabatic heating due to condensation. The implications of the meridional transport of latent heat and of its divergence are discussed in the light of various meteorological considerations. The total mean meridional transport of latent energy is southward in the equatorial regions and northward in middle and high latitude regions whereas the meridional transports associated to transient and standing disturbances are predominantly positive (from south). The contribution of Hadley cell for the total southward transport of latent energy becomes dominant in the lower troposphere of equatorial regions.

RÉSUMÉ — On présente une étude du rôle de la vapeur d'eau dans l'énergétique de la circulation générale de l'atmosphère. La vapeur d'eau est l'absorbant le plus important de l'atmosphère et sa distribution module «l'input» de l'énergie dans le système. En dégageant de la chaleur latente elle va générer l'énergie potentielle disponible zonalle et perturbée. L'énergie disponible des pérturbations est partiellement fournie par le rechauffement non adiabatique due à la condensation de la vapeur d'eau. On étudie d'abord les implications du transport méridional d'énergie latente et de sa divérgence à la lumière de diverses considérations météorologiques.

Le transport méridional total de l'énergie latente est dirigé vers le sud dans les régions équatoriales et vers le nord dans les régions des latitudes

(*) The research reported in this paper has been possible through the suport of the U. S. National Science Foundation under grant Nº G P — 3657.

(**) Received November 30, 1965.

moyennes et élevées, tandis que le transport méridional associé aux pérturbations stationaires et transientes est, d'une façon générale, positive. La contribution des circulations du type Hadley est dominante pour le transport vers le sud de l'énergie latente dans la basse troposphère des régions équatoriales.

1. *INTRODUCTION*

One of the possible approaches to the study of the general circulation of the atmosphere is to examine certain integral requirements deduced from dynamical principles governing the motion of the atmosphere, formulated in therms of physical properties such as energy, momentum, mass, water vapor content, etc.

This approach has been used extensively at the M. I. T. Planetary Circulations Project.

According to the principle of the conservation of mass, water substance cannot be created or destroyed within the atmosphere. The water balance may therefore be taken as a constraint on the general circulation. The necessity for the transport of water in the atmosphere arises from the existence of an excess of precipitation over evaporation in certain regions, with a reversal of prevailing conditions over other areas. Since storage effects of the atmosphere are small enough to be neglected, the excesses and deficits must be made up through the transport of water by atmospheric circulations, since there can be no significant net flux of water into or out of the atmosphere as a whole. Therefore, water vapor can be regarded as an indicator of the mechanisms which maintain the general circulation.

In dealing with the energetics of the atmosphere, one cannot ignore the existence of water component in its various phases. In the vapor phase, it is the most active constituent of the atmosphere with regard to radiative processes. It is a highly selective absorber of incoming solar energy; and also an important emitter of long wave radiation. The solid and liquid phases in the form of clouds have a profound influence upon the spatial distribution of planetary albedo, and consequently upon the amount of solar energy that is available for absorption by the earth. Furthermore clouds influence the long wave radiative balance, and through this, the vertical distribution of temperature in the atmosphere.

Since all its phases can occur within the usual range of the observed temperatures there are large amounts of energy associated with the phase changes that play an important part in the energy budgets of the earth and of the atmosphere.

With the imposed horizontal flux of water vapor is associated a transport of energy in the form of latent heat which constitutes an important part of the energy balance of the atmosphere. The corresponding vertical transport serves to compensate for radiative effects which tend to cool the atmosphere as a whole (1) (2). Finally, the energy associated with phase changes alters the baroclinicity of the atmosphere thereby influencing the kinetic energy, momentum and vorticity fields.

The present paper intends to give some aspects of the results obtained in the study of the water balance requirements of the atmosphere, and their implications for the energetics and the mechanisms of the maintenance of the general circulation.

2. *NOTATIONS AND FORMULAE*

In the present study we shall use the following notations:

λ = longitude
φ = latitude
p = pressure
t = time
a = radius of the earth
$u = a \cos \varphi \, d\lambda/dt$ = eastward wind component
$v = a \, d\varphi/dt$ = northward wind component
$\mathbf{v} = u \, \mathbf{i} + v \, \mathbf{j}$ = horizontal wind vector
$\omega = dp/dt$ = «vertical velocity»
z = height of an isobaric surface
g = acceleration of gravity
$\Phi = gz$ = geopotential
α = specific volume of the air
ρ = density of the air
T = temperature
R = gas constant
c_p, c_v = specific heats at constant pressure and at constant volume
$k = R/c_v$
$\theta = T_p^{-k} p_o^k$ = potential temperature
q = specific humidity
L = latent heat of condensation
$T_e = T + \frac{L}{c_p} q$ = equivalent temperature

ν = zenith angle of the sun

$dm = a^2 \cos \varphi \, d\lambda \, d\varphi \frac{dp}{g}$ = mass element

P = precipitation

E = evaporation

$U = c_v \, T$ = internal energy

$H = c_p \, T$ = enthalpy

$\Phi^* = U + \Phi$ = total potential energy

Ω = angular velocity of the earth

$f = 2 \, \Omega \, sen \, \varphi$ = Coriolis parameter

$\mathbf{F}$ = frictional force $= F_\lambda \mathbf{i} + F_\varphi \mathbf{j}$

$\dot{Q}_F = \frac{dQ_F}{dt}$ = heating rate due to conduction and friction

$\dot{Q}_R = \frac{dQ_R}{dt}$ = heating rate due to radiation

$\dot{Q}_L = \frac{dQ_L}{dt}$ = heating rate due to condensation

$u = g^{-1} \int_o^{po} dp$ = seccional mass

$W = g^{-1} \int_o^{po} q \, d \, p$ = precipitable water

$\mathbf{Q} = g^{-1} \int_o^{po} q \, \mathbf{v} . \, dp = Q_\lambda \mathbf{i} + Q_\varphi \mathbf{j}$ = water vapor vector transport

$Q_\lambda = g^{-1} \int_o^{po} q \, u \, . \, dp$ = zonal transport of water vapor

$Q_\varphi = g^{-1} \int_o^{po} q \, v \, . \, dp$ = meridional transport of water vapor

S_q = source function for water vapor

Λ = availability of energy

$\Gamma = - \frac{\partial T}{\partial z}$ = lapse rate

$\Gamma_d = - \frac{g}{c_p}$ = dry adiabatic lapse rate

$\mathfrak{P} = P_i^j = - \mu \left[\frac{2}{3} \delta_i^j \frac{\partial v^k}{\partial x^k} - \left(\frac{\partial v^i}{\partial x^j} - \frac{\partial v^j}{\partial x^i} \right) \right]$ = Navier-Stokes tensor

$\mathfrak{R} = R_i^j = \overline{v_i' \, v'^j}$ = Reynolds tensor

$$\overline{x} = \tau^{-1}\int_o^\tau x\, dt = \text{time average of } x$$

$$x' = x - \overline{x} = \text{deviation from time average}$$

$$[x] = (2\,\pi)^{-1} \oint x\, d\lambda = \text{zonal average of } x$$

$$x^* = x - [x] = \text{deviation from zonal average}$$

$$\overset{m}{x} = \pi^{-2} \int\int^{\pi/2} x\, d\lambda\, d\varphi = \text{hemispheric average of } x \text{ over an isobaric surface}$$

$$x'' = x - \overset{m}{x} = \text{deviation from hemispheric average}$$

$$<x> = \frac{1}{sen\,(\varphi_j + 10) - sen\,\varphi_j} \int\int x \cos\varphi\, d\varphi\, d\lambda = \text{space average for the latitudinal belt } \varphi_j$$

$$[x\,y]_E = [\overline{x'\,y'}] + [\overline{x}^*\,\overline{y}^*] = \text{total eddy covariance of } x \text{ and } y$$

$$[\overline{x'\,y'}] = \text{transient eddy covariance of } x \text{ and } y$$

$$[\overline{x}^*\,\overline{y}^*] = \text{standing eddy covariance of } x \text{ and } y$$

$$K_M = \frac{1}{2}\int ([\overline{u}]^2 + [\overline{v}]^2)\, dm = \text{zonal kinetic energy}$$

$$K_E = \frac{1}{2}\int ([\overline{u'^2} + \overline{v'^2}] + [\overline{u}^{*2} + \overline{v}^{*2}])\, dm = \text{eddy kinetic energy}$$

$$\gamma = \left\{ \begin{array}{l} -\alpha\left(\dfrac{1}{\theta}\dfrac{\partial\overset{m}{\theta}}{\partial p}\right)^{-1} \\ \left(\overset{m}{T} - \dfrac{p}{R}\, c_p \dfrac{\partial \overset{m}{T}}{\partial p}\right)^{-1} \\ \left(\dfrac{T}{\theta}\right)^2 \dfrac{\overset{m}{T}^{-1}\, g}{(\Gamma_d - \Gamma)} \end{array} \right\} = \text{static stability parameter}$$

$$A_M = \frac{c_p}{2}\int \gamma\, [\overline{T}]''^2\, dm = \text{zonal available potential energy}$$

$$A_E = \frac{c_p}{2}\int \gamma\, ([\overline{T'^2}] + [\overline{T}^{*2}])\, dm = \text{eddy available potential energy}$$

$$D(K) = \int \mathbf{v} \,.\, \mathbf{F}\, dm = \text{rate of frictional dissipation of kinetic energy due to small scale turbulence and to eddy stresses at the boundary}$$

$G(A)$ = rate of generation of available energy due to non--adiabatic effects

$C(K_E, K_M)$ = rate of conversion from eddy kinetic energy to zonal kinetic energy by the eddy momentum transport

$C(A_M, A_E)$ = rate of conversion from zonal to eddy available potential energy by the eddy sensible heat transport

$C(A_M, K_M)$ = rate of conversion from zonal available potential energy to zonal kinetic energy by mean meridional circulations

$C(A_E, K_E)$ = rate of conversion from eddy available potential energy into eddy kinetic energy by large-scale eddy processes

$\nabla = \mathbf{i}\,\frac{\partial}{a\cos\varphi\,\partial\lambda} + \mathbf{j}\,\frac{\partial}{a\,\partial\varphi}$ = surface spherical gradient operator on an isobaric surface

In this discussion the primitive hydrostatic equations for the atmosphere are written as follows:

a) equations of motion

$$\frac{\partial u}{\partial t} + \mathbf{v}\,.\,\nabla u + \omega\frac{\partial u}{\partial p} - \left(f + \frac{u\,tg\,\varphi}{a}\right) + \frac{1}{a\cos\varphi}\frac{\partial\Phi}{\partial\lambda} - F_\lambda = 0 \qquad (1)$$

$$\frac{\partial v}{\partial t} + \mathbf{v}\,.\,\nabla v + \omega\frac{\partial v}{\partial p} + \left(f + \frac{u\,tg\,\varphi}{a}\right) + \frac{1}{a}\frac{\partial\Phi}{\partial\varphi} - F_\varphi = 0 \qquad (2)$$

b) equation of hydrostatic equilibrium

$$\frac{\partial\Phi}{\partial p} + \frac{R}{p}\,T = 0 \qquad (3)$$

c) equation of continuity

$$\frac{\partial\omega}{\partial p} + div_p\,\mathbf{v} = 0 \qquad (4)$$

d) equation of continuity for the water vapor

$$\frac{\partial q}{\partial t} + div_p\,q\,\mathbf{v} + \frac{\partial}{\partial p}\,q\,\omega = \frac{dq}{dt} \equiv S_q \equiv -\frac{\dot{Q}_L}{L} \qquad (5)$$

e) equation of the first law of thermodynamics

$$\frac{\partial \theta}{\partial t} + \mathbf{v} \cdot \nabla \theta + \omega \frac{\partial \theta}{\partial p} - \frac{\theta}{T} \frac{(\dot{Q}_F + \dot{Q}_R + \dot{Q}_L)}{c_p} = 0 \qquad (6)$$

3. *THE WATER VAPOR IN THE GLOBAL BALANCE OF TOTAL ENERGY OF THE ATMOSPHERE*

3.1. *The balance equation of global energy*

The atmosphere contains significant amounts of potential energy (gravitational), internal energy (heat), latent energy (heat of condensation) and kinetic energy of various scales of motion.

For a moist atmosphere the total amount of potential energy, Φ, the internal energy, U, and the latent energy, LW, in a unitary column of the atmosphere in a state of hydrostatic equilibrium is proportional to the mean equivalent temperature of the column or to the wheighted potential equivalent temperature

$$\begin{aligned} \Phi + U + LW &= g^{-1} \int c_p \, T_e \, dp \\ &= p_o^{-k} \, g^{-1} \int c_p \, p^k \, \theta_e \, dp \end{aligned} \qquad (7)$$

where W is the precipitable water content of the column, L the latent heat of condensation, assumed to be constant and Θ_e the potential equivalent temperature. This total energy may be designated as *total moist potential energy*.

For a dry atmosphere the total amount of potential energy and internal energy for a unitary column of the atmosphere in a state of hydrostatic equilibrium is proportional to the total enthalpy of the column or to the wheighted potential temperature, θ:

$$\begin{aligned} \Phi + U &= g^{-1} \int c_p \, T \, dp \\ &= p_o^{-k} \, g^{-1} \int c_p \, p^k \, \theta \, dp \end{aligned} \qquad (8)$$

Since the generation and the destruction of both forms of energy (potential plus internal) occurs simultaneously it is costumary to consider the two forms of energy as a single form, the so called *total potential energy*, Φ^*.

MARGULES (1903) has firmly established that the maintenance of the atmospheric motions in a synoptic scale against the dissipation is due to the conversion of total potential energy into kinetic energy. The rate of generation of total potential energy (internal plus potential), which has to be resupplied depends upon non adiabatic heating including radiation, frictional heating, the release of latent heat, heating of contact of the atmosphere with the earth (transport of sensible and latent heat by turbulent difusion), etc.

The mechanism of conversion is basically a sinking of colder air and a rising of warmer air at same level. It is then required an horizontal gradient of temperature for the process to continue. Thus only a small fraction of the total potential energy is really available for conversion into kinetic energy of the actual atmospheric motions. The process of generating *available potential energy* is essentially through the heating of warm regions and the cooling of cooler regions at the same isobaric level which is equivalent to a local decrease of entropy.

The local balance equation for the total energy which expresses the conservation of total energy for the atmosphere, may be written (3), (4), (5) in the form:

$$\frac{\partial}{\partial t}\rho(U+\Phi+K)+div\left[\rho(c_pT+Lq+\Phi+K)\mathbf{v}-(\mathfrak{R}+\mathfrak{p})\cdot\mathbf{v}\right]=\rho\dot{Q} \qquad (9)$$

where $\mathfrak{p}$ is the Navier-Stokes viscosity tensor, $\mathfrak{R}$ is the Reynolds turbulence tensor.

If this equation is integrated over the volume of a polar cap τ, bounded by a wall Σ, the resulting equation after averaged in time over the period considered becomes:

$$\frac{\partial}{\partial t}\iiint_\tau \rho\overline{(U+\Phi+K)}\,d\tau+\iint_\Sigma \rho\overline{(c_pT+Lq+K+\Phi)\,v_N}\,d\Sigma-$$
$$-\iint_\Sigma\left[\overline{(\mathfrak{R}+\mathfrak{p})\cdot\mathbf{v})}\right]_N d\Sigma=\iiint_\tau \rho\,\overline{\dot{Q}}\,d\tau \qquad (10)$$

The local variation of the total energy in the polar cap results from:

a) the flux of total energy in the form of enthalpy $(\overline{c_p T})$, of latent heat $(\overline{Lq})$, of potential energy $(\Phi=\overline{gz})$ and of kinetic energy of existing motions $\left(\frac{\rho\overline{\mathbf{v}^2}}{2}\right)$, across the boundary Σ;

b) the flux of energy due to the action of frictional forces, which would ordinarily consist of a dissipation due to molecular viscosity and to small-scale turbulence which can produce no significant tangencial stresses;

c) a production of energy due to non-adiabatic heating ($\dot{Q}$).

It must be pointed out that kinetic energy of existing motions is very small compared with the other forms of energy.

3.2. *The meridional flux of moist latent energy*

Latent heat is one of the component of the flux of energy in equation (10). Analyses of the transport fields of water vapor can be regarded, in fact, as representations of the fluxes of latent heat. As was discussed in previous papers (4), (6), (7) the total mean horizontal flux of latent heat above a point on the earth's surface is given by

$$\overline{L\mathbf{Q}} = g^{-1}\int \overline{Lq\,\mathbf{v}}\,dp = L\,(\overline{Q}_\lambda\,\mathbf{i} + \overline{Q}_\varphi\,\mathbf{j}) \tag{11}$$

where the latent heat of condensation, L, is assumed to be constant.

From the hemispheric analyses of the quantities $\overline{Q}_\lambda$ and $\overline{Q}_\varphi$ (6) the values of the latter quantity have been computed and are shown in TABLE I. By comparing these values with those given by HOU-

TABLE I

Zonally averaged values of the total latent energy transport across latitude circles, $[L\,\overline{Q}_\varphi]$, for the Northern Hemisphere in units of 10^{14} cal/sec. The lower numbers give the component of the total due to Mean Meridional cells $Lg^{-1}\int [\bar{q}]\,[\bar{v}]\,dp$.

Latitude	80°	70°	60°	50°	45°	40°	30°	20°	10°	0°
Winter	— 0,06	0,38	1,34	2,69	3,02	3,22	2,29	— 2,21	— 8,62	— 5,59
		0,05	0,20	0,25	0,40	0,70	— 0,17	— 4,42	— 10,03	— 5,74
Summer	— 0,07	0,29	1,46	4,08	4,15	3,38	1,27	— 1,10	1,15	5,45
		— 0,22	0,05	1,66	1,85	1,29	— 1,26	— 3,98	0,16	5,60
Year	— 0,07	0.32	1,42	3,19	3,48	3,28	1,75	— 1,62	— 3,70	0,00
		— 0.13	— 0,08	0,76	0,58	0,64	— 0,61	— 4,26	— 4,66	0,07

GHTON (1) and by BUDYKO (8), (9), (10) one can see that in some regions water vapor contributes more than 0,30 of the meridional heat flux required to maintain the radiative balance.

The total transport of latent energy can be accomplished by the mean circulations and by the large transient and standing perturbations of the general circulation (11). In order to find the relative contribution of the various processes, the transport components $\overline{qu}$ and $\overline{qv}$ at a given isobaric level may be expanded according to the generalised REYNOLDS scheme:

$$[\overline{qu}] = [\overline{q}]\ [\overline{u}] + [\overline{q}^*\ \overline{u}^*] + [\overline{q'u'}] \tag{12}$$

$$[\overline{qv}] = [\overline{q}]\ [\overline{v}] + [\overline{q}^*\ \overline{v}^*] + [\overline{q'v'}] \tag{13}$$

Thus we may write for the components of the mean zonal and meridional total latent heat transports, respectivelly, the following expressions:

$$L\,[\overline{Q}_\lambda] = L\,g^{-1}\int_o^{po}[\overline{q}]\,[\overline{u}]\,dp + L\,g^{-1}\int_o^{po}[\overline{q}^*\,\overline{u}^*]\,dp + L\,g^{-1}\int_o^{po}[\overline{q'u'}]\,dp \tag{14}$$

$$L\,[\overline{Q}_\varphi] = L\,g^{-1}\int_o^{po}[\overline{q}]\,[\overline{v}]\,dp + L\,g^{-1}\int_o^{po}[\overline{q}^*\,\overline{v}^*]\,dp + L\,g^{-1}\int_o^{po}[\overline{q'v'}]\,dp \tag{15}$$

The terms of these equations are associated with the mean advection of latent heat ($[\overline{q}]\ [\overline{u}]$) and with the mean meridional circulation ($[\overline{q}]\ [\overline{v}]$); with the standing large scale horizontal eddies ($[\overline{q}^*\,\overline{u}^*]$; $[\overline{q}^*\,\overline{v}^*]$) and finally with transient horizontal eddies ($[\overline{q'u'}]$); ($[\overline{q'v'}]$).

In the study of the atmosphere and the earth's energy budgets the meridional transport $L\,\overline{Q}_\varphi$ plays a much more important role than the zonal transport $L\,\overline{Q}_\lambda$. Therefore we will discuss in detail the behaviour of all the components of the meridional transport, $L\,\overline{Q}_\varphi$, and its latitudinal distribution.

The zonally averaged values of the mean total moist latent energy transport across latitude circles, $[\overline{L\,Q_\varphi}]$, for the Northern hemisphere are given in TABLE I. The vertical distribution of the zonally averaged values of meridional transport of latent energy at various latitudes is presented in TABLE II. The extreme northward and southward values occur in the low troposphere in the layer 1000/850 mb.

TABLE II

Zonally averaged values of mean meridional transport of latent energy $g^{-1}[\overline{qv}]$ in units of cal/(mb. cm. sec.) for yearly data at specified latitudes. The levels are given in millibars.

Latitude	70°	60°	50°	45°	40°	30°	20°	10°	0°
1000 mb	6,02	18,06	36,12	41,54	32,51	— 9,63	— 76,45	— 38,53	6,02
850	— 2,41	15,65	38,53	39,13	37,93	4,21	— 17,46	— 7,22	2,41
700	2,41	15,65	28,90	28,90	26,49	1,20	1,81	— 14,45	— 4,82
500	— 0,60	4,21	6,62	6,02	5,42	5,42	3,61	— 0,60	— 3,01

The meridional transient eddy flux of moist latent energy, $Lg^{-1}\int_{o}^{po}[\overline{q'v'}]\,dp$, as shown in TABLE III, is predominantly positive

TABLE III

Zonally averaged values of the mean total meridional transient eddy transport of latent energy $g^{-1}L\int[\overline{q'v'}]\,dp$, in units of 10^{14} cal/sec for yearly and seasonal data at specified latitudes.

Latitude	70°	60°	50°	45°	40°	30°	20°	10°	0°
Winter	0,28	0,85	2,02	2,24	2,28	2,00	1,43	1,22	0,13
Summer	0,46	1,42	2,38	2,26	1,97	1,22	1,15	0,80	— 0,17
Year	0,41	1,30	2,58	2,81	2,45	1,74	1,55	0,86	— 0,07

(northward) over the northern hemisphere. It shows a yearly maximum which occurs near 47,5°N, shifting to the north in summer and to the south in winter. The maximum observed is clearly associated with the mean position of the polar front, as to be expected, in view of the role of the baroclinic perturbations in the eddy meridional transport. The transient eddy transport varies with the altitude and reaches a maximum in the middle latitude region in the lower troposphere near the 850 mb level (TABLE IV).

TABLE IV

Zonally averaged values of the mean zonal transient eddy transport of latent energy, $g^{-1}L\int(\overline{q'v'})\,dp$, in units of cal/(mb. cm. sec) for yearly data at specified latitudes. The levels are given in millibars.

Latitude	65°	55°	45°	35°	25°	15°	5°
1000 mb	— 1,81	— 0,00	— 3,01	— 1,81	+ 6,02	+ 9,63	+ 4,21
850	+ 2,41	+ 3,01	— 4,21	— 5,42	+ 1,81	+ 9,03	+ 7,22
700	+ 0,00	+ 0,00	— 4,21	— 10,84	— 6,02	+ 5,42	+ 13,24
500	+ 4,21	+ 3,61	— 2,41	— 7,22	— 8,43	— 3,61	+ 1,20

The total meridional transport of latent energy associated with the standing-eddies, $L\,g^{-1}\int[\bar{q}^*\bar{v}^*]\,dp$, (TABLE V) is always positive

TABLE V

Zonally averaged values of the mean total meridional standing eddy transport of latent energy $g^{-1}L\int_0^{p_0}[\bar{q}^*\bar{v}^*]\,dp$ in units of 10^{14} cal/sec for yearly and seasonal data at specified latitudes.

Latitude	70°	60°	50°	45°	40°	30°	20°	10°	0°
Winter	0,05	0,30	0,42	0,38	0,24	0,46	0,77	0,19	0,02
Summer	0,04	0,01	0,06	0,05	0,12	1,31	1,73	0,17	0,01
Year	0,04	0,20	0,12	0,10	0,18	0,61	1,09	0,14	0,00

(from south) and in general presents a well defined maximum at 20°N, associated with semi-permanent subtropical anticyclones and another maximum, much less intense, near 55°N associated with the semi--permanent lows prevailing in this region. The vertical distribution of the meridional standing eddy flux of latent heat (TABLE VI) shows that a maximum occurs in the neighbourhood of 22,5°N at 850 mb and another around 55°N near the surface. It is interesting to point

TABLE VI

Zonally averaged values of the mean meridional standing eddy transport of latent energy, $g^{-1}L\left[\bar{q}^{*}\,\bar{v}^{*}\right]$, in units of cal/(mb. cm. sec) for yearly data at specified latitudes. The levels are given in millibars.

Latitude	70°	60°	50°	45°	40°	30°	20°	10°	0°
1000 mb	+ 1,81	+ 4,21	+ 3,01	+ 1,20	+ 2,41	+ 4,82	+ 6,62	+ 2,41	+ 2,41
850	+ 1,20	+ 1,81	+ 1,81	+ 1,81	+ 3,61	+ 6,02	+ 9,63	+ 3,01	+ 1,20
700	+ 0,00	+ 2,41	+ 1,81	+ 0,60	+ 1,20	+ 2,41	+ 4,82	+ 3,01	+ 0,00
500	+ 0,60	+ 0,00	+ 0,00	+ 0,00	+ 0,60	+ 1,20	+ 2,41	+ 1,20	+ 0,00

out that the lowest values occur at 45°N where the largest values of the transient eddy transport of latent heat are observed.

The latitudinal distribution of the total eddy meridional flux (TABLE VII) presents a bimodal distribution, resulting from the

TABLE VII

Zonally averaged values of the mean total meridional eddy transport of latent energy $g^{-1}L\int\left\{\left[\overline{q'\,v'}\right]+\left[\bar{q}^{*}\,\bar{v}^{*}\right]\right\}\,dp$, in units of 10^{14} cal/sec for yearly and seasonal data at specified latitudes.

Latitude	70°	60°	50°	45°	40°	30°	20°	10°	0°
Winter	0,33	1,15	2,44	2,62	2,52	2,46	2,20	1,41	0,15
Summer	0,50	1,43	2,44	2,31	2,09	2,53	2,88	0,97	—0,16
Year	0,45	1,50	2,70	2,91	2,63	2,35	2,64	1,00	—0,07

combination of the latitudinal distribution due to transient eddies $(L\,g^{-1}\int_{o}^{po}[\overline{q'\,v'}]\;dp)$ and to standing eddies $(L\,g^{-1}\int_{o}^{po}[\bar{q}^{*}\,\bar{v}^{*}]\;dp)$ associated with the quasi-permanent features of the atmosphere circulation.

Thus, we can conclude that the effect of the standing eddies is of greatest significance in low latitudes, where the quasi-stationary disturbances are dominant. At middle latitudes the vigorous transient eddies predominate and the standing eddies play a minor role in the meridional transport of energy. However, at 60ºN their importance increases again, especially in winter when semi-permanent lows are most intense.

The comparison of the values of total meridional transport of latent energy and those of the total eddy transport offers the important indirect evidence of the existence of the three-cell regime with two direct cells and one indirect cell. The values of the mean meridional transport of latent energy by the mean meridional circulations are shown for comparison in TABLE I.

The contribution of the Hadley cell for the total southward transport of latent heat in the equatorial region becomes dominant, whereas the contribution of the other two cells play a minor role in the process, the eddies being the major factor in the total meridional flux of latent heat.

3.3. *The water vapor and the generation of total potentital energy*

The rate of non-adiabatic heating, $\dot{Q}$, due to conduction and friction, $\dot{Q}_F$, to radiation, $\dot{Q}_R$, and to condensation, $\dot{Q}_L$, will be written *in extenso* by adding up the individual contributions corresponding to the different physical processes that participate in the total heat balance.

We shall use the operator $<(\quad)>$ to define the mean value of a quantity within a zone j of the atmosphere which extends from latitude φ_j to latitude $\varphi_j + 10^{\circ}$.

Then, the mean rate of heating of the atmosphere $< \dot{Q}(z) >$ is given by

$$< \dot{Q}_j > = < \overline{S}_j(\infty) \downarrow > + < \overline{G}_j(0) \uparrow\downarrow > + < \overline{L}_j(0) \uparrow > + \\ + < \overline{C}_j(0) \uparrow > - < \overline{S}_j(0) \downarrow > - < \overline{G}_j(\infty) \uparrow > \qquad (16)$$

where $< \overline{S}_j(z) >$ and $< \overline{G}_j(z) >$ are the intensities of the solar and long wave radiation respectively, and $< \overline{L}_j(0) \uparrow >$ and $< \overline{C}_j(0) \uparrow >$ are the latent heat and sensible heat transport at the lower boundary respectively. The arrows indicate the direction of the net flux.

We will proceed to show that all the terms are influenced direct or indirectly by the presence of water vapor in the atmosphere.

The quantity of radiant energy absorbed and scattered by the atmosphere at each point of the globe, $< \overline{S}_j(\infty) \downarrow > - < \overline{S}_j(0) \downarrow >$, is a function of the air mass, $u = g^{-1} \int dp$, and the precipitable water, $W = g^{-1} \int q \, dp$. According to HOUGHTON this quantity is given by:

$$< \overline{S}_j(\infty) \downarrow > - < \overline{S}_j(0) \downarrow > = 0{,}175 \, (\overline{W.u})^{0{,}39} \cos \nu \;\; (cal/cm.^2 min) \qquad (17)$$

where ν is the zenith angle of the sun.

To show the dependence of long wave radiative balance, $< \overline{G}_j(0) \uparrow\downarrow > - < \overline{G}_j(\infty) \uparrow >$, upon water vapor content, one need only refer to any radiation chart (see, for instance, Elsasser radiation chart).

Studies of precipitable water content such as those published by STARR, PEIXOTO and CRISI (12) have importante application to investigation of radiation and heat balance in the atmosphere. The maps of precipitable water may be used to find the spatial distribution of time averaged solar energy absorption. Furthermore, from the spatial distribution of specific humidity at different levels, one can examine the three dimensional distribution of this effect. These maps, in conjunction with temperature analyses could thus be useful in the computation of long wave absorption and emission at a given point in the atmosphere. Many specific applications of infrared radiation technology, however, require instantaneous information concerning atmosphere moisture.

Let us analyse now the effect of the clouds in the disposition of the solar radiation.

The planetary albedo has a mean value of 0.34, with a minimum of 0.28 in the subtropical regions, which are relatively devoid of cloudiness, and a maximum of 0.67 in the polar regions due to the presence of snow cover (1). Hence, the latitudinal distribution of solar energy available for absorption has a maximum in the subtropical regions, around 20°N, as shown by BUDYKO *et al.* (9). However, because the zonally averaged amount of precipitable water in the atmosphere is a monotonically decreasing function of latitude (6) and clouds are not important as absorbers of solar energy, the atmospheric absorption does not show this maximum. When the earth and atmosphere are taken as a system, the subtropical maximum is still evident, though it is suppressed by the effects mentioned above.

In order to obtain the distribution of $<\overline{L}_j(0)\uparrow>$ one can combine the values $<\overline{P-E}>$ as obtained by various authors (12) with values of evaporation $<\overline{E}>$, such as those given by BUDYKO *et al.* (8), (9), so as to obtain the distribution of $<\overline{P}>$ (*). The latter, when multiplied by the proper constant, yields $<\overline{L}_j(0)\uparrow>$. The values of $<\overline{P-E}>$ can be obtained from the divergence of the water vapor transport field $\overline{\mathbf{Q}}$, as has been discussed on several occasions (4), (12).

In fact for a unit column of air extending from the earth's surface (pressure p_o) to the top of the atmosphere (pressure $p=0$), the water vapor balance equation can be written:

$$\frac{\partial \overline{W}}{\partial t} + div\ \overline{\mathbf{Q}} = \overline{S}_q \tag{18}$$

where $\overline{S}_q$ represents the net source of water substance in the atmospheric column. The source and sinks of water vapor in the atmosphere are due primarly to evaporation E from the surface of the earth and to precipitation P. For all pratical purpose $\overline{S}_q$ is given by the excess of evaporation over precipitation, $\overline{E-P}$. Thus taking the time average for the given time period (one year), the equation for atmospheric water vapor balance becomes:

$$\frac{1}{a\ cos\ \varphi}\left[\frac{\partial \overline{Q}_\lambda}{\partial \lambda} + \frac{\partial}{\partial \varphi}\left(\overline{Q}_\varphi\ cos\ \varphi\right)\right] = \overline{S}_q \equiv (\overline{E-P}) \tag{19}$$

because for this time interval $\frac{\partial \overline{W}}{\partial t}$ may be taken as zero.

The values of the water vapor transport field $\mathbf{Q} = (Q_\lambda \mathbf{i} + Q_\varphi \mathbf{j})$ have been discussed and computed (4), (6), (7) and the analysis of

(*) At this point one might raise the objections that, since the author consulted the work of BUDYKO to obtain values of $<\overline{E}>$, why did he not use the same source to obtain values of $<\overline{P}>$ directly, instead of going through the rather involved procedure described above. Justification for the procedure used rests on the fact that evaporation is a smoother function of space and time (almost monotonically decreasing function of latitude) than is precipitation and hence the former is more adaptable to the averaging techniques used in this type of study.

the distribution of the mean total horizontal divergence, $\nabla \cdot \overline{\mathbf{Q}}$, for the years of 1950 and 1958 have been already studied (12), (14), (15). The analysis of $\nabla \cdot \overline{\mathbf{Q}}$ show the existence of divergence centers alternating with convergence centers and exhibit considerable detail.

The divergence by ten degree latitude belts has been computed (12), using the expression:

$$< \overline{\nabla \cdot \mathbf{Q}} > = \frac{1}{a \cos \varphi} \frac{\partial}{\partial \varphi} \oint \overline{Q}_{\varphi} \cos \varphi \, d\lambda = < \overline{E - P} > \qquad (20)$$

The zonal values show a strong meridional variation with negative values ($\overline{E - P} < 0$) in the equatorial and middle latitude regions and with positive values ($\overline{E - P} > 0$) in the subtropical latitudes, where the divergence field shows a belt of maximuma. From the previous relation it is obvious that, over long periods, evaporation must exceed precipitation in these regions.

Thus the subtropical regions always act as a source of moisture for the atmosphere as a whole, while the equatorial, middle and high latitude regions act primarly as sinks.

3.4. *The hydrologic cycle in the global energetics of the atmosphere*

Now let us consider the expression of the rate of non-adiabatic heating and the latitudinal variation of its various components.

The values of $< \overline{S}_j(\infty) \downarrow > - < \overline{S}_j(0) \downarrow >$, which measure the rate of absorption of solar energy by the atmosphere and the values of the longwave radiation emitted by the earth and the atmosphere $< \overline{G}_j(\infty) \uparrow >$ were gruperted and discussed by HOUGHTON (1). Both are slowly varying functions of latitude. The difference between these quantities $\{ < (\overline{S}_j(\infty) \downarrow > - < \overline{S}_j(0) \downarrow) > - < \overline{G}_j(\infty) \uparrow > \}$ which appears in equation (16) is still more uniform.

According to BUDYKO $< \overline{C}_j(0) \uparrow >$ is always positive, smaller than any of the other terms in (16), and slowly varying with latitude; also BUDYKO's given values of $< \overline{G}_j(0) \uparrow \downarrow >$ are practically constant with respect to latitude (8), (9). In contrast to these slowly varying functions, the values of $< \overline{L}_j(0) \uparrow >$ determined by the latitudinal distribution of precipitation have a much more pronounced latitudinal variation with a maximum over equatorial regions, a secondary maximum over high latitudes and minima over polar and subtropical regions (14), (15).

Hence we conclude that the latitudinal distribution input of energy into the atmosphere through non-adiabatic heating which is the generation source for all the potential energy and responsable for the maintenance of atmospheric motions against dissipation is essentially modulated by the function $\mathscr{L}(\varphi) \equiv < \overline{L}_i(0) \uparrow >$ as can be inferred from the inspection of equation (16). Thus the hydrological cycle has a profound influence upon the energetics of the general circulation of the atmosphere.

In the *evaporation-condensation-precipitation cycle*, virtually all evaporation takes place at the surface of the earth, and therefore the cooling involved in the process does not directly affect the atmosphere and does not affect the generation of available potential energy. On the other hand, condensation occurs principally within the atmosphere, where the release of latent heat has a direct influence.

Virtually all the condensate will eventually reach the ground in the form of precipitation but there may, of course, be further transports before this occurs. However, it is well established that the transport of water in the vapor phase far exceeds that in the liquid and solid phases in the atmosphere (6). This fact justifies the assumption, used in the present discussion, that precipitation is a measure of condensation. Precipitation, surface drainage and runoff complete the mass cycle, but are unimportant as far as the energetics of the atmosphere are concerned. Viewing the cycle as a whole, we see that it produces a net transfer of heat from the earth's surface to the atmosphere where it modulates the total meridional input of energy.

An assessment of the importance of the hydrological cycle in the energetics of the atmosphere requires the knowledge of geographical distribution of condensation, but not that of evaporation.

Since the hypothesis of evaporation-precipitation *in situ* cannot be accepted there must also be a transfer of heat from one geographical location to another and from one level to another. Thus we see that this cycle plays a part in regulating the temperature distribution of the earth's surface and atmosphere. Moreover, the transport of energy in the form of latent heat can be looked upon as a mechanism which the general circulation uses to adjust itself to the heterogeneous boundary conditions imposed by the differential absorption of solar energy.

The water cycle, from the point of view of mass and its implications on the field of hydrology has been discussed elsewhere (11), (12), (15).

4. *THE MAINTENANCE OF THE MEAN AND EDDY CIRCULATIONS IN THE ATMOSPHERE*

4.1. *Available potential energy and kinetic energy*

The concept of mean available potential energy presented by MARGULES (1903) (16) has been elaborated and discussed later by LORENZ (1955) (17) who was able to express it in terms of the temperature variance on an isobaric surface. The expression of the available potential energy is given by:

$$A = \frac{1}{2} c_p \int \gamma \, \overline{T''^2} \, dm \tag{21}$$

where γ is the stability factor:

$$\gamma = -\alpha \left(\frac{1}{\theta} \frac{\partial \overset{m}{\theta}}{\partial p} \right)^{-1} = \left(\overset{m}{T} - \frac{p}{R} c_p \frac{\partial \overset{m}{T}}{\partial p} \right)^{-1}$$

Through an analysis of variance of the temperature field the available potential energy $A = \frac{1}{2} c_p \int \gamma \, \overline{T''^2} \, dm$ may be partitioned into zonal available potential energy A_M and eddy available potential energy A_E

$$A = A_M + A_E \tag{22}$$

where

$$A_M = \frac{1}{2} c_p \int \gamma \, [\overline{T}]''^2 \, dm \tag{23}$$

and

$$A_E = \frac{1}{2} c_p \int \gamma \, [\overline{T'^2} + \overline{T}^{*2}] \, dm \tag{24}$$

Similarly, through an analysis of variance of the wind field the total kinetic energy $K = \frac{1}{2} \int \mathbf{v}^2 \, dm$ may be partitioned into zonal kinetic energy, K_M, corresponding to zonally averaged motions of the atmosphere and into eddy kinetic energy, K_E, the eddy motions:

$$K = K_M + K_E \tag{25}$$

where:

$$K_M = \frac{1}{2}\int\left\{[\bar{u}]^2 + [\bar{v}]^2\right\}\, dm \tag{26}$$

and

$$K_E = \frac{1}{2}\int\left\{[\overline{u'^2} + \overline{v'^2}] + [\bar{u}^{*2} + \bar{v}^{*2}]\right\}\, dm \tag{27}$$

It is convenient to consider K_M, K_E, A_M and A_E as separate forms of energy. A balance equation for each of these forms can be obtained following the usual and well known procedure. These equations have some terms in common with opposite signs and will be regarded as conversion functions among the various forms of energy (4), with the understanding that we reserve the term conversion functions for the terms which express clearly physical mechanisms that take part in the atmosphere.

4.2. *Balance equations for the kinetic energy*

In the derivation of the balance equation for the zonal kinetic energy the equation (1) of zonal motion is first multiplied by $[\bar{u}]$, next averaged in time and finally integrated over the mass for a polar cap or all for the atmosphere. The resulting equation, expressing the balance between the various processes which take part in the maintenance of K_M, can be written under the simbolic form:

$$\frac{\partial K_M}{\partial t} = C\,(K_E, K_M) + C\,(K_{M,\varphi}, K_{M,\lambda}) + D\,(K_M) + A\,(K_M) + W_e(K_M) \tag{28}$$

where:

a)

$$C\,(K_E, K_M) \approx \int [\overline{u\,v}]_E \cos\varphi \frac{\partial}{a\,\partial\varphi}\left[\frac{[\bar{u}]}{\cos\varphi}\right] dm + \int [\overline{u\,w}]_E \frac{\partial\,[\bar{u}]}{\partial p}\, dm \tag{29}$$

is the conversion from eddy into zonal kinetic energy by horizontal and vertical eddies, which depends upon the transport of angular momentum along the gradient of angular velocity;

b)

$$C\,(K_{M,\varphi}, K_{M,\lambda}) = \int f\,[\overline{u}]\,[\overline{v}]\,dm + \int [\overline{v}]\,[\overline{u}]^2\,\frac{tg\,\varphi}{a}\,dm \qquad (30)$$

is the conversion of energy from the mean meridional motions into mean zonal motions due to the Coriolis effect in an Hadley regime;

c)

$$D\,(K_M) = \int [\overline{u}]\,[\overline{F}]\,dm \qquad (31)$$

is the viscous, turbulent and frictional dissipation;

d)

$$A\,(K_M) = \int\int ([\overline{u}] + [\overline{\omega}])\,\frac{1}{2}\,[\overline{u}]^2\,\frac{d\Sigma}{g} \qquad (32)$$

is the advection of K_M across the boundary Σ by mean meridional overturnings; and finally:

e)

$$W_e\,(K_M) = \int\int [\overline{u}]\,([\overline{u\,v}]_E + [\overline{u\,\omega}]_E)\,\frac{d\Sigma}{g} \qquad (33)$$

is the work performed on the volume τ by the eddy stresses at the boundary Σ.

The balance equation for the eddy kinetic energy is derived following an analogous procedure. The zonal equation of motion is multiplied by $u' + \overline{u}^*$ and the meridional equation by $v' + \overline{v}^*$. After adding them together and averaging in time, the integration in space over the mass of a polar cap will lead to the balance equation:

$$\frac{\partial K_E}{\partial t} = -C\,(K_E, K_M) + C\,(A_E, K_E) + D\,(K_E) + A\,(K_E) + W_p\,(K_E) + W_e\,(K_E) \qquad (34)$$

where:

a)

$C\,(K_E, K_M)$ is the rate of conversion from eddy into zonal kinetic energy;

b)

$C(A_E, K_E)$ is the rate of conversion from eddy available potential into eddy kinetic energy:

$$C(A_E, K_E) = -\int \frac{R}{p} ([\overline{\omega' T'}] + [\overline{\omega}^* \overline{T}^*]) \, dm; \qquad (35)$$

c)

$D(K_E)$ is the dissipation of eddy kinetic energy due to friction:

$$D(K_E) = \int [\overline{u F_\lambda}]_E \, dm + \int [\overline{v F_\varphi}]_E \, dm; \qquad (36)$$

d)

$A(K_E)$ is the advection of eddy kinetic energy through the boundary Σ by mean meridional circulations:

$$A(K_E) = \iint_\Sigma \{[\overline{v}] + [\overline{\omega}]\} \cdot \frac{1}{2} \{[\overline{u^2}]_E + [\overline{v^2}]_E\} \frac{d\Sigma}{g} \qquad (37)$$

and

e)

$W_e(K_E)$ is the work performed on the layer by the eddy stresses at the boundaries.

A net conversion from zonal to eddy kinetic energy would occur if the eddies acted to transfer angular momentum from latitudes of high angular velocity to those of low angular velocity in the manner of large scale viscosity. However, observations indicate that such a conversion does not take place (18), (20), but instead eddy kinetic energy is converted into zonal kinetic energy and appears to be the main source for the maintenance of the zonal currents against dissipation by turbulence, by friction, etc.

4.3. *Balance equations for the available potential energy*

The balance equation for the available potential energy can be derived from the equation (6) of first law of thermodynamics when it is multiplied by $\gamma [\overline{\theta}]''$ where γ is the stability parameter and resolving the potential temperature into the various eddy components $\theta'' = [\overline{\theta}]'' + \overline{\theta}^* + \theta'$.

The resulting equation after averaged in time and next integrated in space for a polar cap or for all the atmosphere and neglecting the triple correlations, assumes for the zonal available potential energy the form:

$$\frac{\partial A_M}{\partial t} = - C(A_M, A_E) - C(A_M, K_M) + G(A_M) \tag{38}$$

This equation expresses that there is a balance between the various processes:

a) The conversion $C(A_M, A_E)$ from zonal into eddy available potential energy by horizontal and vertical eddy processes:

$$C(A_M, A_E) = - c_p \int \Upsilon \left\{ [\overline{v' T'}] + [\bar{v}^* \bar{T}^*] \right\} \frac{\partial [\bar{T}]}{a\, \partial \varphi} dm - \\ - c_p \int \Upsilon \left(\frac{T}{\theta} \right) \left\{ [\overline{\omega' T'}] + [\bar{\omega}^* \bar{T}^*] \right\} \frac{\partial [\bar{\theta}]''}{\partial p} dm \tag{39}$$

which depends upon the horizontal and vertical eddy transports of sensible heat $[\overline{v\,T}]_E$ and $[\overline{\omega\,T}]_E$ along the gradient of temperature. When eddies transport sensible heat against the temperature gradient (from warm to cold zones) there is a conversion from zonal available potential energy A_M into eddy available potential energy A_E in the manner of large scale conductivity.

b) The conversion $C(A_M, K_M)$ from zonal available potential energy into zonal kinetic energy by mean meridional circulations:

$$C(A_M, K_M) = - R \int p^{-1} [\bar{\omega}]'' [\bar{T}]'' dm \tag{40}$$

c) The generation $G(A_M)$ of zonal available potential energy by non-adiabatic heating:

$$G(A_M) = \int \Upsilon [\bar{T}]'' [\bar{Q}]'' dm \tag{41}$$

The low latitude regions with a warm troposphere are continuously heated by the surplus of incoming solar radiation over outgoing terrestial long wave radiation, whereas middle and high latitude regions with lower temperatures are cooled by the same radiation energy balances.

This results in a large positive value of the covariance $\widetilde{[T]'' [Q]''}$ and consequently in a large generation of zonal available potential energy.

The equation of balance for the eddy available potential energy can be derived multiplying the equation of the first law of thermodynamics by $\gamma(\overline{\theta}^* + \theta')$ where γ is the static stability factor, next averaged with respect to time and then integrated in space. The final equation will be:

$$\frac{\partial A_E}{\partial t} = C(A_M, A_E) - C(A_E, K_E) + G(A_E) \qquad (42)$$

where $G(A_E)$ is the generating function of eddy available potential energy and is given by:

$$G(A_E) = \int \gamma [\overline{T' Q'} + \overline{T}^* \overline{Q}^*] \, dm \qquad (43)$$

and the other terms have the previous meaning.

As was mentioned in § 3.1 only a small part of the total potential energy is converted into kinetic energy in the atmosphere. The maximum possible value is the available potential energy. However, it may now be inferred that the principal via of conversion of available potential energy to kinetic energy is a conversion of eddy available potential into eddy kinetic energy accomplished by downward eddy motions of cooler air and upward eddy motion of warmer air.

4.4. *The energy cycle of the general circulation in the troposphere*

We are led to the following scheme of the energy cycle of the general circulation in the troposphere. The net heating of the atmosphere in low latitudes and the net cooling in high latitudes result in a continual generation of zonal available potential energy. The atmosphere is baroclinically unstable and virtually all this energy is converted into eddy available potential energy by the resulting eddies. Some of this energy may be dissipated in the eddies through the combined effects of radiation, condensation, evaporation, the heat flux near the ground and the heating of colder portions of the eddies and the cooling of warmer portions; the remainder is partly converted into eddy kinetic energy through the sinking of colder air and the rising of warmer air in the eddies.

Some of the kinetic energy in the large scale eddies is dissipated in a cascade regime by generating smaller and smaller eddies and by friction; the remaining part of this energy is converted into kinetic energy for the zonal currents.

Most of the zonal kinetic energy is dissipated by turbulence and by friction; a small residual is converted into zonal available potential energy again by an indirect meridional circulation and this brings back to the beginning of the cycle. Schematically we accept that the energy cycle in the troposphere proceeds from A_M to K_M, through the following scheme:

$$A_M \longrightarrow A_E \longrightarrow K_E \longrightarrow K_M \longrightarrow A_M$$

It therefore appears that eddies play a crucial role in regulating the general circulation. It is this very basic fact that has laid down the foundation for the modern concepts on general circulation and has changed all the perspective of the dynamics of the atmosphere (STARR, 1958) (20).

5. *THE WATER VAPOR AND THE ENERGY CYCLE OF THE GENERAL CIRCULATION*

5.1. *Availability in a moist atmosphere*

The formulation of the energy cycle, as has been presented, does not incorporate directly the water component in the atmosphere. The fundamental reason lays in the difficulty in defining a *reference state* for such complex system as the moist atmosphere; the specification of the reference state is essential for the assessment of the *availability* of the energy of a system. The reference state has to be a *dead state*, inert for any thermodynamical transformation, characterized by the most stable state of thermodynamical equilibrium, without local contrast in entropy, suposed to have the maximum possible value compatible with the constraints imposed to the total system, and so with a minimum total potential energy.

The availability measures the maximum value of the *useful work*, W_u, corresponding to a state of a subsystem within an ideal atmosphere in a dead state and is denoted by Λ. Thus:

$$\Lambda \equiv \Phi^* - \Phi^*_{min} \tag{44}$$

It follows that for any state of any atmospheric subsystem in the ideal dead atmosphere

$$\Lambda \geqslant 0 \ .$$

For the most stable state of the subsystem, which is then a dead state,

$$\Lambda = 0$$

which corresponds to the identification of the subsystem with the ideal atmosphere. For a finite change from the subsystem in state a to the subsystem in state b

$$W_u = \int_{\widehat{ab}} dW \leqslant - \int_a^b d\Phi \tag{45}$$

or

$$W_u \leqslant \Phi^*_a - \Phi^*_b = \Delta \Lambda^b_a \tag{46}$$

It follows that it is impossible to transform into useful work (kinetic energy of the atmospheric motions) all the variation of the availability when the subsystem undergoes any transformation that brings it from a thermodynamic state to another.

The behaviour of the water component in the atmosphere makes it very difficult to define a «dead state» for the moist atmosphere. With the possible changes of phase there is a mass transfer from one phase to another within the atmospheric subsystem and eventually a transfer of mass out of the total system (the atmosphere) through precipitation. The atmosphere is an open system for the water component and, furthermore, its content in water vapor is not even statistically constant. All these processes and transformations alter profoundly the time and the spatial distribution of energy and entropy within the system and makes it extremely difficult to define and to find the balance of those quantities. Thus, any model of reference state for this system so complex has to be highly idealized and restrictive, involving rather gross simplifications and limitations when compared with the real moist atmosphere.

A possible reference state for a moist monophasic and trivariant atmosphere, useful perhaps under very simple conditions, would be that corresponding to an ideal atmosphere with a minimum total moist potential energy and with uniform isobaric distributions of temperature, entropy and moisture content, as given, for example, by the corresponding isobaric averaged values.

5.2. *Effects of water vapor on the general circulation*

In spite of the mentioned difficulties in taking into account explicitly the influence of the water component in the present theory of the general circulation, we can infer some effects of the water vapor on the energy cycle formulated for a dry atmosphere and discuss some implications due to the presence of water vapor in the atmosphere.

The rate of generation of total potential (internal plus potential) energy depends upon the total non-adiabatic heating. However, the generation of available potential energy is determined by the spatial distribution of non-adiabatic heating with respect to the temperature field. Zonal and eddy available potential energies are affected directly by the water vapor through absorption of solar radiation and the processes of long wave radiation, through the changes in the albedo and, finally, through condensation and, to a less extent, through evaporation.

Let us consider the heat energy stored in the moist atmosphere due to water vapor. The existence of the energy which we shall denote as *moist «available» potential energy* can play a part in the dynamics of the atmosphere only if it is converted into available potential energy, through the process of a phase change (mainly through condensation). Since the hemispheric distribution of water vapor is almost axially symetric with a well defined meridional gradient, there is in the atmosphere a large storage of *total moist zonal «available» potential energy*. The distribution has, of course, a non symetric component, which suggest the concept of *total moist eddy «available» potential energy*.

The rate of generation of available potential energy, $G(A)$, is proportional to the covariance of non-adiabatic heating and temperature. Thus, through the release of latent heat the water vapor plays a direct role in the production of zonal and eddy available potential energy, $G(A_M)$ and $G(A_E)$, as can be inferred from their corresponding equations (41) and (43).

It follows also from the mathematical expressions of the conversion terms of the various forms of energy, as presented in § 4, that the water vapor plays an indirect role in the conversion of the various forms of energy because the water vapor can introduce significant changes in the u, v, ω and T and γ fields.

The time averaged transport of latent heat by both transient and standing eddies is poleward at all latitudes as it is shown in the corresponding tables of § 4. Since the mean zonal temperature also decreases poleward (21), (22), this means that the eddies must, on the average, be converting latent zonal available potential energy into latent eddy available potential energy, $C(A_M, A_E) > 0$. We should expect this effect to be most intense in middle and high latitudes where there is a maximum of eddy activity. To the extent that at these latitudes condensation occurs predominantly near the frontal zones there is a generation of eddy available potential energy — that is, some of the moist available potential energy released — in the perturbations along the frontal zone. This is precisely where the conversion of available potential energy into kinetic energy, through baroclinic processes, is taking place.

In fact condensation $(\dot{Q}_L > 0)$ in middle latitude perturbations occurs generally with southerly warm ascending currents $(T' > 0)$, $(\omega < 0)$ and evaporation $(\dot{Q}_L < 0)$ with northerly cold subsiding currents $(T' < 0)$, $(\omega > 0)$. Since the correlation between $\dot{Q}_L$ and T is positive there is a generation of eddy available potential energy with the release of latent heat, $\{G(A_E)\}_L > 0$. However, when the radiation effects are considered the oposite takes place. The moist warm air from south $(T' > 0)$ in its movement towards higher latitudes is cooled $(\dot{Q}_F + \dot{Q}_R < 0)$, while the northerly cold air $(T' < 0)$ is warmed $(\dot{Q}_R + \dot{Q}_F > 0)$. In this case the covariance along the latitude circles between T and $\dot{Q}_R$ is negative which leads to a destruction of eddy available potential energy, $\{G(A_E)\}_R < 0$. The net value for $G(A_E)$ will depend upon the balance of these oposite effects; $G(A_E)$ is presumably negative (18), but it might happen that the release of latent heat could alter at times the sign of $G(A_E)$.

In these disturbances the covariance between T and ω is negative $(\overline{[\omega\, T]}_E < 0)$ and there is a conversion of eddy available potential energy into eddy kinetic energy, $C(A_E\, K_E) > 0$. The condensation process reinforces the vertical motion field by heating warm rising air, thus augmenting the rate of generation of eddy kinetic energy by baroclinic processes. AUBERT (23) has shown the importance of this effect.

It is also interesting to point out that, since the large-scale condensation and evaporation processes are accompanied simultaneously by rising of warm air and sinking of cold air, respectively, the available energy so generated is not exposed to the dissipation through long wave radiation as it would happen, with the potential energy generated by the transport of sensible heat. Probably the efficiency of the conversion of available potential energy generated through the release of latent heat into kinetic energy is very high.

The strong precipitation observed in equatorial regions due to the Hadley cell contributes decisively to the production of zonal available potential energy because the release of latent heat occurs in regions where the temperature is already higher than average and almost zonally uniform ($[\overline{T}]''[\overline{Q}_L]'' > 0$). It might appear, at first glance, that the secondary maximum in the curve of $\mathscr{L}(\varphi) \equiv < \overline{L}_j(0) \uparrow >$ at high latitudes is associated with the destruction of zonal available potential energy, because heating is taking place in a region where the zonally averaged temperature is relatively lower. However, in this case, the zonally averaged picture is misleading, for, as we have seen above, the heating due to the release of latent heat at these latitudes occurs selectively in the warm air giving rise, *de facto*, to the generation of eddy available potential energy, $G(A)_E > 0$. When we take the zonal average of the rate of generation of available potential energy, we find, in fact, that there is a relative maximum in these latitudes.

Let us consider still another aspect of the energy cycle where the effects of water vapor must be taken into account. All energy conversion processes involving available potential energy are dependent upon static stability. We note that static stability occurs in the denominators of equations (21), (23), (24), (39), (41), (43). The direction of conversion between available potential energy and kinetic energy depends upon the sign of this term, and the rate depends upon its numerical value, all other things being considered constant. If, initially, the reaction proceeded so as to generate kinetic energy, then eventually, the rising of warm air and sinking of cold air involved in the process, if unopposed by other factors, would stabilize the atmosphere, thus bringing the reaction to a halt. However, the release of latent heat alters this situation in the following manner. The condensation process occurs mainly near the 700 mb level, which is below the level of maximum rate of energy conversion. Hence, the release of latent heat destabilizes the atmosphere in the regions where kinetic energy is being generated, thus contributing to the perpetuation of the conversion processes, because the dominant baroclinic waves become more unstable and the release of latent heat leads to an acceleration of their growth.

The adiabatic lapse rate Γ_d that figures in the stability parameter γ in various equations is applicable only in an unsaturated environment. In a saturated atmosphere it must be replaced by Γ_s, if the pseudo-adiabatic assumption is to be used. Hence, in a saturated atmosphere the rate of energy generation and conversion will have to be adjusted by the factor:

$$\frac{\Gamma_s}{\Gamma_d}\frac{(\Gamma-\Gamma_d)}{(\Gamma-\Gamma_s)}$$

5.3. *Heating of the atmosphere due to condensation*

A possible functional representation for $\dot{Q}_L$ can be obtained from the balance equation for the water component. At a given isobaric level p, the rate of heating, ${}_p\dot{Q}_L$, due to the condensation of dq grams of water vapor is given by:

$${}_p\dot{Q}_L = -L\frac{dq}{dt} \tag{47}$$

Using the water vapor continuity equation we can write the previous equation, averaging in time:

$${}_p\overline{\dot{Q}}_L = -L\left(\frac{\partial\bar{q}}{\partial t} + \overline{\mathbf{v}\cdot\nabla q} + \overline{\omega\frac{\partial q}{\partial p}}\right) \tag{48}$$

This equation transformed with the equation of continuity can be written as follows:

$${}_p\overline{\dot{Q}}_L = -L\left(\frac{\partial\bar{q}}{\partial t} + \nabla\cdot\overline{q\,\mathbf{v}} + \frac{\partial\overline{q\omega}}{\partial p}\right) \tag{49}$$

We should also include in this equation the vertical eddy difusion due to small scale turbulence. However, in large scale processes this effect can be neglected.

The vertical integration of this equation leads to the equation of the divergence of total water vapor transport referred in the previous paragraph:

$$\int_{p}^{p_0} {}_p\dot{Q}_L \, dp \equiv \overline{\dot{Q}_L} = - L \frac{\partial \overline{W}}{\partial t} - L \, div \, . \, \overline{\mathbf{Q}} \tag{50}$$

because the contribution of the terms in ω and q, due to the boundary conditions is zero.

The analysis of the divergence of water vapor transport, proportional to the heating associated with the release of latent heat, shows centers of convergence ($div \, . \, \overline{\mathbf{Q}} < 0$) alterning with centers of divergence ($div \, . \, \overline{\mathbf{Q}} > 0$) over all the northern hemisphere (12). Therefore the spatial distribution the heating of the atmosphere due to the release of latent heat is not uniform. This illustrates the need for considering, in all problems involving the generation of available potential energy, the actual spatial covariance of the divergence of water vapor and temperature, rather than the covariance of the zonally averaged values of the respective fields.

With the usual REYNOLDS expansion equation (49) will assume the form:

$${}_p\dot{Q}_L = - L \left(\frac{\partial \bar{q}}{\partial t} + \nabla \cdot \overline{q' \mathbf{v}'} + \frac{\partial \, \overline{\omega' q'}}{\partial p} + \nabla \cdot \bar{q} \bar{\mathbf{v}} + \frac{\partial \, \bar{\omega} \bar{q}}{\partial p} \right); \tag{51}$$

In a (λ, φ, p, t) coordinate system this equation is written:

$$\begin{aligned} {}_p\dot{Q}_L &= - L \left(\frac{\partial \bar{q}}{\partial t} \right) - L \frac{1}{a \cos \varphi} \left(\frac{\partial \, \overline{q}}{\partial \lambda} + \frac{\partial \overline{q v} \cos \varphi}{\partial \varphi} + \frac{\partial \, \overline{\omega q}}{\partial p} \right) \\ &= - L \frac{\partial \bar{q}}{\partial t} - L \frac{1}{a \cos \varphi} \left(\frac{\partial \, \overline{u' q'}}{\partial \lambda} + \frac{\partial \, \overline{q' v'}}{\partial \varphi} \cos \varphi + \frac{\partial \, \overline{\omega' q'}}{\partial p} \right) - \\ &\quad - L \frac{1}{a \cos \varphi} \left(\frac{\partial}{\partial \lambda} \bar{u} \bar{q} + \frac{\partial}{\partial \varphi} \bar{q} \bar{v} \cos \varphi + \frac{\partial}{\partial p} \bar{\omega} \bar{q} \right) \end{aligned} \tag{52}$$

In the long time average the local change $\frac{\partial \bar{q}}{\partial t}$ is very small and can be disregarded in comparison with the other terms. All the other terms, except $\frac{\partial \, \overline{q' \omega'}}{\partial p}$ and $\frac{\partial \, \bar{\omega} \bar{q}}{\partial p}$ can be computed from hemispheric humidity charts such as those already published (6).

The local values of the divergence associated with the transport of water vapor by the transient and standing eddies, $\nabla \cdot \overline{q' \mathbf{v}'}$, and $\nabla \cdot \bar{q}\, \bar{\mathbf{v}}$ are of the same order of magnitude. However, the mass integral for all the atmosphere gives in general higher values for the divergence associated with the standing eddies ($\nabla \cdot \overline{q}^* \overline{\mathbf{v}}^*$) than those associated with the divergence of transient eddies ($\nabla \cdot \overline{q' \mathbf{v}'}$). The values of the divergence, $\nabla \cdot \bar{q}\, \bar{\mathbf{v}}$, decrease rapidly with the altitude because $\bar{q}$ becames very small as the height incrases. The values of $\nabla \cdot \overline{q' \mathbf{v}'}$ decrease also with height, with a maximum at 850 mb level.

The evaluation of the vertical divergence given by the terms $\frac{\partial \overline{q' \omega'}}{\partial p}$ and $\frac{\partial \bar{q}\, \bar{\omega}}{\partial p}$ depends on the knowledge of the ω-field at various levels. The values of ω could be obtained from the observations using the method suggested by BARNES (24). For a given layer of the atmosphere the vertical divergence of water vapor may have values comparable to those of the horizontal divergence (and perhaps of oposite sign). However, for all the atmosphere, since the integration has to be done in p, the horizontal divergence field will predominate in view of the boundary values of $\bar{q}$ and $\bar{\omega}$ at the bottom and at the top of the atmosphere.

5.4. *Generation of available potential energy due to release of latent heat*

Since the effects of water vapor are so complex we shall only discuss here the effects of phase changes of water vapor in the atmosphere. Simultaneously a method for computing the rate of generation of available potential energy due to release of latent heat wil be presented.

The generation of available potential energy due to the release of latent heat is obtained substituing $\dot{Q}$ by the value of $\dot{Q}_L$, as given by equation (51), in the expressions (41) and (43):

$$G(A_M) = \int \gamma\, [\overline{T}]''\, [\overline{Q}]''\, dm$$

$$G(A_E) = \int \gamma\, [\overline{T' Q'} + \overline{T}^* \overline{Q}^*]\, dm$$

The rate of generation of zonal available potential energy due to the release of latent heat is therefore:

$$G(A_M)_L = -L\int\gamma\,[\overline{T}]''\frac{\partial\,[\bar{q}]''}{\partial t}\,dm - L\int\gamma\,[\overline{T}]''\,\nabla\cdot[\overline{q\,\mathbf{v}}'']\,dm -$$
$$- L\int\gamma\,[\overline{T}]''\frac{\partial\,[\overline{q\,\omega}]''}{\partial p}\,dm \qquad (53)$$

The rate of generation of eddy available potential energy will be given by:

$$G(A_E)_L = -L\int\gamma\left[\overline{T'\frac{\partial q'}{\partial t}}\right]dm - L\int\gamma\,[\overline{T'\,\nabla\cdot(q\mathbf{v})'}]\,dm -$$
$$- L\int\gamma\left[\overline{T'\frac{\partial\,(q\,\omega)'}{\partial p}}\right]dm - L\int\gamma\left[\overline{T}^*\frac{\partial\bar{q}^*}{\partial t}\right]dm -$$
$$- L\int\gamma\,[\overline{T}^*\,\nabla\cdot\overline{q\mathbf{v}}^*]\,dm - L\int\gamma\left[\overline{T}^*\frac{\partial\overline{q\,\omega}^*}{\partial p}\right]dm \qquad (54)$$

For computation purposes it is more convenient to write the expressions of $G(A_M)_L$ and of $G(A_E)_L$ in the (λ, φ, p, t) coordinate system as we did above.

In principle all the terms of $G(A_M)_L$ and of $G(A_E)_L$ due to the release of latent heat could be computed by using time series of spatial isobaric distributions (charts) of the fields of T, q and of $\overline{q\,\mathbf{v}} \equiv (\overline{q\,u}\,\mathbf{i} + \overline{q\,v}\,\mathbf{j} + \overline{q\,\omega}\,\mathbf{k})$. Through the analysis of these charts with a suitable grid point the calculations could be made using the standard finite differences method.

In longe time average the local change $\frac{\partial\bar{q}}{\partial t}$ is very small and can be neglected. However, for short time intervals, it may become significant, and cannot be disregarded in the evaluation of the covariances.

The contribution of the terms $\left[\overline{T'\frac{\partial\,(\omega\,q)'}{\partial p}}\right]$ and $\left[\overline{T}^*\frac{\partial\,(\overline{q\,\omega})^*}{\partial p}\right]$ for all the atmosphere is proportional to the difference of the surface covariances of $\overline{q\,\omega}$ and $\overline{T}$ over the boundary surfaces Σ_1 and Σ_2 of the atmosphere. In fact we may write for the last term:

$$\left[\overline{T}^*\frac{\partial\,(\overline{\omega q})^*}{\partial p}\right] = \left[\frac{\partial}{\partial p}(\overline{T}^*\,(\overline{\omega\,q})^*)\right] - \left[(\overline{\omega\,q})^*\frac{\partial\,\overline{T}^*}{\partial p}\right] \qquad (55)$$

The second term on the right hand side may be neglected because we accept as a good approximation that

$$\frac{\partial T}{\partial p} \simeq \frac{[T]}{\partial p} \tag{56}$$

For all the atmosphere, since $dm = g^{-1}\, dp\, d\Sigma$, the resulting contribution will be:

$$\iiint \left[\overline{T}^* \frac{\partial \overline{(\omega p)}^*}{\partial p} \right] dm = \iiint_o^{po} \frac{\partial}{\partial p} [\overline{T}^* (\overline{\omega q})^*] \frac{dp}{g}\, d\Sigma =$$
$$= g^{-1} \iint_\Sigma [\overline{T}^* \, \overline{\omega q}^*]\, d\Sigma \Big|_o^{po} =$$
$$= g^{-1} \iint_{\Sigma_1} [\overline{T}^* \, \overline{\omega q}^*]\, d\Sigma_1 - g^{-1} \iint_{\Sigma_2} [\overline{T}^* \, \overline{\omega q}^*]\, d\Sigma_2 \tag{57}$$

The values of these covariances are presumably very small in view of the boundary conditions at the bottom and at the top of the atmosphere for both quantities $\overline{q}$ and $\overline{\omega}$.

It is expected that this suggested method for computating $G(A_E)_L$ will lead to good estimates of the rate of generation of eddy available potential energy in extratropical regions due to release of latent heat. In low latitudes the estimates are not so good, because the mesoscale phenomena, so important for the vertical transport of water vapor in this region, have not been taken into account.

6. *FINAL COMMENTS*

We have seen that water vapor plays a vital role in the energetics of the general circulation. It is the most important absorber of solar energy in the atmosphere, and hence, its distribution influences the form of the energy input into the system. The release of latent heat constitutes another important energy input. Through this process the water vapor distribution significantly influences the motions, and motions, in turn, deform the water field. This complex feedback mechanism constitutes what is probably the most important non-adiabatic effect in the general circulation.

To illustrate its importance let us consider the highly simplified situation where a mean meridional cell has formed in response to the temperature difference between the air at two latitudes. Let us examine what happens if there is evaporation at the surface below the descending part of the cell, a transport of water vapor by the lower branch, and precipitation in the area of ascent. In a crude qualitative sense, this corresponds to the Hadley cell at low latitudes. The release of latent heat due to the moisture transport by the cell increases the temperature difference between the two latitudes, thus increasing the strength of the motions. In effect, the rising air is being warmed by the latent heat release, and is thus forced to rise more rapidly. Continuity demands that the entire cell be strengthened by this effect.

Until recently the many dynamical models which have been put forth in an attempt to explain the features of the general circulation have been formulated for a dry atmosphere. This approach has been necessitated by the difficulties which arise when one tries to describe analytically the mechanism which transport water vapor. Very close to the ground (in the lowest few meters) microscale effects predominate, while at higher levels, mesoscale motions affect the vertical transports, and horizontal transports are accomplished predominantly by macroscale motions. Further complications arise when one attempts to specify the necessary and sufficient conditions for the condensation process to take place. The subsequent precipitation process is also extremely difficult to describe, even if a pseudoadiabatic process is assumed. Although some of these difficulties have been dealt with sucessfully in a recent model put forth by the staff of the Geophysical Fluid Dynamics Laboratory, of ESSA (Environmental Sciences Services Administration), Washington (25), the understanding of the role of the water vapor in the dynamics of the atmosphere has not yet reached the stage where any single model can accurately simulate all the above processes (26).

ACKNOWLEDGMENTS — The author is grateful to Professor VICTOR P. STARR for his continuous interest, encouragement and advice, to Dr. BARRY SALTZMAN for his fruitful comments and discussions; and to Dr. JOHN WALLACE for his valuable suggestions and criticisms and for his most helpful cooperation and substantial contributions throughout the course of this study. He is indebted to Com. PEDRO MAGALHÃIS for translating from russian some of the articles referred in this study. Acknowledgment is made of support from U. S. National Science Foundation in the performance of this research at M. I. T.

REFERENCES

(1) HOUGHTON, H. G. — Jour. of Meteo., **11**, 1, 1954.
(2) SUTCLIFFE, R. C. — Quart. J. R. Met. Soc., **82**, 385, 1956.
(3) STARR, V. P. — *Compendium of Meteorology*. Amer. Met. Soc., Boston, 1954.
(4) PEIXOTO, J. P. — Revista Faculdade Ciências de Lisboa, 2.ª série B — VI, 1958.
(5) SALTZMAN, B. — Jour. of Meteo., **14**, 66, 1957.
(6) PEIXOTO, J. P. — *Sci. Rpt.* No. 3, *General Circulation Project*, M. I. T., 1958.
(7) STARR, V. P., PEIXOTO, J. P. and LIVADAS, G. C. — Geof. Pura e Appl., **39**, 174, 1958.
(8) BUDYKO, M. I. — *Atlas of heat balance*, Leningrad, 1955.
(9) BUDYKO, M. I., BERYLAND, I. G. and ZUBENCK, L. I. — Bull. Acadm. Sc., U. S. R. R., Geog. Serv., **3**, 17, 1955.
(10) BUDYKO, M. I. — A. I. M. — UGGI, Toronto, 1957.
(11) STARR, V. P. and PEIXOTO, J. P. — Arch. f. Met., und Biokli., **14**, 111, 1964.
(12) STARR, V. P., PEIXOTO, J. P. and CRISI, A. — Tellus, **17**, 4, 1965.
(13) STARR, V. P. and WHITE, R. M. — *Final Report, Part* 2, General Circulation Project, M. I. T., 1954.
(14) STARR, V. P. and PEIXOTO, J. P. — Tellus, **10**, 189, 1958.
(15) PEIXOTO, J. P. — Revista Faculdade Ciências de Lisboa, 2.ª série B — VII, 25, 1959.
(16) MARGULES, M. — Jahrb. Kaiserlich-Koniglichen Zentralanstalt Meteor., 1903.
(17) LORENZ, E. N. — Tellus, **7**, 157, 1955.
(18) OORT, A. H. — Tellus, **16**, 309, 1964.
(19) NEWELL, R. E. — Jour. Geophys. Res., 68, 3949, 1963.
(20) STARR, V. P. — Jour. Meteor. Soc. of Japan, **36**, 167, 1958.
(21) PEIXOTO, J. P. — *Sci. Rpt.* No. 4, *Planetary Circulations Project*, M. I. T., 1960.
(22) PEIXOTO, J. P. — Prize «Artur Malheiros» of the Academy of Sciences of Lisbon, 1960.
(23) AUBERT, E. — M. I. T., Ph. D. Thesis, Dep. of Meteo., M. I. T., 1957.
(24) BARNES, A. A. — *Final Report* — C. No. AF19 (604) — 5223, *Planetary Circulations Project*, M. I. T., 1963.
(25) MANABE, S., SMAGORINSKI, J. and STRICKLER, R. — *«Simulated Climatology of a General Circulation Model with a Hidrologic Cycle»* — Geophysical Fluid Dynamics of ESSA, Washington 1965 (to be published in Month. Wea. Review).
(26) PEIXOTO, J. P. — Memórias da Academia das Ciências de Lisboa, vol. X, 1966.