SCALING, EXTENDED VECTOR MESON DOMINANCE AND THE EXISTENCE OF THE NEUTRAL PION (*)

R. VILELA MENDES

Instituto de Fisica e Matemática,
Av. Gama Pinto Lisbon-Portugal

ABSTRACT — It is shown that in the framework of the extended vector meson dominance model, scaling in electron-positron annihilation is incompatible with the finiteness of the $\pi^0 \rightarrow 2\gamma$ decay width.

The implications of the result for the validity of the extended vector dominance idea or for scale breaking are discussed.

RESUMÉ — On montre que dans le cadre du modèle generalisé de dominance des mesons vectoriels, l'invariance d'échelle dans l'annihilation electron-positron n'est pas compatible avec le charactère fini du taux de déclin $\pi^0 \rightarrow 2\gamma$.

On discute les implications du résultat en ce qui concerne la validité du modèle de dominance vectoriel generalisé ou la violation de l'invariance d'échelle.

The purpose of this note is to show that the following three statements cannot simultaneously be true propositions:

1. The $\pi^0 \rightarrow 2\gamma$ decay width is finite.
2. The one-photon contribution to $e^+ e^- \rightarrow$ hadrons scales as $1/s$ for large s.
3. All photon-hadron couplings, both at low and at high energies, may be approximated by an extended vector meson dominance (EVMD) model.

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The first two statements are self explanatory. As for the third, it is meant to state the hypothesis that the virtual photon mass dependence, of the spectral functions of unsubtracted mass dispersion relations associated to photon-hadron vertices, can be approximated by a sum of 1^- pole contributions, ranging over an infinite spectrum (which for simplicity we will take to be discrete).

By «all photon-hadron couplings» it is meant that the EVMD hypothesis should apply both to single and multiple-photon processes. However, to deal with multiphoton processes in the framework of an EVMD model, one needs information on the strengths of vertices involving several vector mesons. Hence, our assignment of a precise meaning to proposition 3 is completed by requiring such couplings to follow the same general pattern as the couplings of excited physical states in the dual resonance model (DRM).

In the Veneziano model or in a relativistic quark model [1] the squared masses of the vector mesons obey a linear mass formula $m^2(n) \sim m_0^2(a_1 + a_2 n)$. Here no such restriction will be made. In fact, proposition 3 only requires the function $m^2(n)$ to be increasing for large n and unbounded, for otherwise the EVMD model would be unable to describe the spectral functions in the asymptotic high energy limit.

The proof of the result goes as follows:

In the framework of the EVMD model, the one photon contribution to the total $e^+ e^-$ annihilation cross section into hadrons is:

$$\sigma(s) = \frac{16 \pi^2 \alpha^2}{s^{3/2}} \sum_n \frac{m_n^3}{f_n^2} \frac{m_n \Gamma_n}{(s - m_n^2)^2 + m_n^2 \Gamma_n^2} \tag{1}$$

where the sum runs over an infinite spectrum of vector resonances and as in the usual VMD equation, we have neglacted the non-diagonal overlap functions of vector meson amplitudes. The coupling of the n-th resonance to the photon is $e\, m_n^2/f_n$, and Γ_n is its total width.

Define a function $F(s)$ related to the cross section $\sigma(s)$ by:

$$F(s) = \frac{s^{3/2} \sigma(s)}{16 \pi^2 \alpha^2}.$$

To study the asymptotic behaviour of $F(s)$ one considers convergence in the mean of the series in Eq. (1) to a smooth function $c/s^{k-3/2}$. For each interval $(m_{n+1}^2 - m_{n-1}^2)/2$ there is a new resonance giving a contribution $2\pi m_n^3/f_n^2$ to the integral of $F(s)$. On the other hand, using the mean value theorem, one writes the integral of $c/s^{k-3/2}$ in the same interval as:

$$(2) \qquad \frac{c}{(m(n+\alpha))^{2k-3}} \times \frac{m_{n+1}^2 - m_{n-1}^2}{2} = \frac{c}{(m(n+\alpha))^{2k-3}} \times \frac{\partial}{\partial n} m^2(n+\alpha')$$

where $|\alpha|, |\alpha'| < 1$.

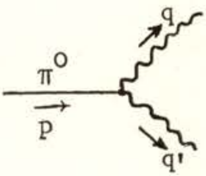
Equating (2) with the resonance contribution one obtains for large n

$$(3) \qquad \frac{1}{f_n^2} \propto \frac{1}{m_n^{2(k-1)}} \frac{\partial}{\partial n} \log m_n^2 \qquad (n > \mathrm{N})$$

If $\sigma(s)$ scales as $1/s$, then $k = 1$ and one has: [2]

$$(4) \qquad \frac{1}{f_n^2} \propto \frac{\partial}{\partial n} \log m_n^2 \qquad (n > \mathrm{N})$$

Let us now consider the $\pi^0 \rightarrow 2\gamma$ vertex with the pion on the mass shell and general off-shell photon lines



The vertex amplitude is

$$(5) \qquad \mathrm{T}_{\mu\nu}(q, q') = \varepsilon_{\mu\nu\alpha\beta}\, q^\alpha q'^\beta\, \mathrm{T}(p^2, q^2, q'^2).$$

In the finite mass dispersion relation (FMDR) approach [3] to the study of $\mathrm{T}(p^2, q^2, q'^2)$ one disperses in q^2 with p^2 and q'^2 fixed. Then, the usual assumption is that the low mass region is dominated by a few resonances whereas the high mass region is light-cone (LC) dominated. In this case only the contributions of a finite number of vector mesons are taken into account and the corrections to VMD should include not only the light cone contribution but also «screening factors» that subtracted from the pole contributions eventually make the particle form factors vanish faster than $1/q'^2$ (so as to preserve LC-dominance of the asymptotic behaviour).

In an EVMD approach (with an infinite number of poles) the point of view should be quite different, for if such a model is to have full merit it should be to the FMDR approach as dual models are to finite energy sum rules (FESR). I. e., the amplitudes should be described exclusively by a sum of infinitely many pole contributions, with all its relevant properties, including the asymptotic behaviour, determined by the rate of variation of the coefficients in such a series. That such an approach does not lead into any obvious contradictions for single-photon processes is clear from our calculation of the $1/f_n^2$ behaviour (Eq. (3)) as well as from recent calculations by other authors [4] on EVMD descriptions of deep annihilation and deep inelastic structure functions. The straightforward generalization of this «dual» EVMD point of view to multiphoton processes is equivalent to an hypothesis of saturation of *unsubtracted* mass dispersion relations by an infinite number of pole contributions. For our results it will not be even necessary that such an approximation be a very good one, in fact, it is enough not to be infinitely bad, i. e not to require infinite corrections.

One then writes:

$$T(p^2, q^2, q'^2) = \frac{1}{\pi} \int \frac{d\xi}{\xi - q^2 - i\varepsilon} \operatorname{Im} T(p^2, \xi + i\varepsilon, q'^2). \tag{6}$$

In the neighbourhood of the V_n pole

$$\operatorname{Im} T(p^2, \xi + i\varepsilon, q'^2) \simeq \pi\, \delta(\xi - m_n^2)\, G_{\pi V_n \gamma}(q'^2)\, e\, m_n^2 / f_n .$$

Hence $T(p^2, q^2, q'^2)$ becomes

$$T(p^2, q^2, q'^2) \simeq \sum_n \frac{e\, m_{\rho_n}^2\, G_{\pi\rho_n\gamma}(q'^2)}{f_{\rho_n}(m_{\rho_n}^2 - q^2)} + \frac{e\, m_{\omega_n}^2\, G_{\pi\omega_n\gamma}(q'^2)}{f_{\omega_n}(m_{\omega_n}^2 - q^2)} + \cdots \tag{7}$$

where the dots stand for finite width corrections.

Writting now dispersion relations for the $\pi V_n \gamma$ vertices and applying once more the EVMD hypothesis:

$$\mathrm{T}(p^2, q^2, q'^2) \approx 2 \sum_{n,n'} \frac{e^2 m^2_{\rho_n} m^2_{\omega_{n'}} g_{n,n'}}{f_{\rho_n} f_{\omega_n} (m^2_{\rho_n} - q^2)(m^2_{\omega_{n'}} - q'^2)} + \cdots . \tag{8}$$

The coupling constants $g_{n,n'}$ are the part of the factorized residues of $G_{\pi V_n \gamma}(q'^2)$ at the $m^2_{V_{n'}}$ poles that correspond to the $\pi \omega_n \rho_{n'}$ vertices. The n-dependence of these coupling constants is a critical point in the present calculation. Let us assume for the moment (see discussion below) that at least the diagonal constants $g_{n,n} \equiv g_n$ do not vanish when $n \to \infty$ {i. e. $g_n \geq g$ for almost all $n > N$} [5].

Using Eq. (3) one may now study the convergence properties of the series of Eq. (8). For any fixed q^2 and q'^2 and for n large the remainder of the diagonal ($n = n'$) subseries is

$$\sum_{n>N} \frac{g}{f_n^2} \propto \sum_{n>N} \frac{1}{m_n^{2(k-1)}} \frac{\partial}{\partial n} \log m_n^2 . \tag{9}$$

For $k = 1$ the r. h. s. of Eq. (9) is greater or equal to

$$\int_N^\infty dn \frac{\partial}{\partial n} \log m_n^2 = \log m_n^2 \Big|_{n=N}^{n \to \infty}$$

and if m_n^2 is unbounded the series will be divergent. On the other hand if $k > 1$ the series will be convergent even for m_n^2 unbounded.

In conclusion: if the proposition 2. is violated (i. e. $k > 1$) the program of simple pole dominance by an infinite number of vector mesons does not run here into any contradictions. On the other hand if one requires both $k = 1$ and m_n^2 unbounded then the pole dominance hypothesis of unsubtracted dispersion relations would predict an infinite $\pi^0 \to 2\gamma$ decay width.

In the derivation above, use was made of the hypothesis that g_n does not vanish when $n \to \infty$. In fact it is hard to understand how it could be otherwise in any dynamical model with an infinite sequence of vector mesons. In a relativistic quark model [1], for example, ω_n and ρ_n share the same Bethe-Salpeter amplitudes and differ from each other only on their SU(3) properties which do not depend on n. Then it would be hard to understand how the coupling constant for a virtual transition from a ρ_n to a ω_n with the emission of a space-symmetric state (the pion) could have a strong n-dependence.

To obtain a better quantitative feeling of the pattern of couplings to be expected in a model with an infinite number of resonances I have computed, in the DRM, a vertex involving one scalar ground state particle and two excited transverse physical states with one photon index at the levels N and N′. The coupling strength for such a vertex is:

$$< -\pi, N, \varepsilon_i \,|\, v_0(k'', 1) \,|\, \pi', N', \varepsilon_{i'} > = (2\pi)^4 \delta^4(-\pi - k'' + \pi')\, G$$

where the states of momentum $-\pi$ and π' are transverse states at the levels N and N′ created from the vacuum by the transverse operator $A_{i,n}$ of Del Giudice, Di Vecchia and Fubini [6] and $v_0(k'', 1)$ is the ground state vertex operator. For the general non-collinear case the result is:

$$G = \delta_{N,N'}\, \delta_{i,i'} - \frac{2\,(\vec{k}'' \cdot \vec{\varepsilon}_i)\,(\vec{k}'' \cdot \vec{\varepsilon}_{i'})}{\sqrt{N N'}} .$$

One sees that for large N, N′ the diagonal couplings (N=N′) tend to a constant value whereas the non-diagonal couplings become very small. If this is the pattern of couplings that one should expect in a model of extended vector dominance then the critical assumption of non-vanishing of g_n when $n \to \infty$ is justified and the proof goes through in this respect.

On the other hand, if one insists that the high mass vector mesons do decouple after all, then that is the same as to say that something becomes anomalous at the high masses in the EVMD model. This is tantamount to say that the model is necessarily unreliable at high energies for multiple photon processes, i. e., that proposition 3 is not really a true proposition.

Notice that the use of the DRM to compute the mass dependence of strong interaction vertices is quite consistent with proposition 2, for as shown by Greco [7], in a recent paper, in an EVMD description of scaling in e. m. interactions one is quite naturally led to require scaling for the strong interactions and such a behaviour has been shown to hold in dual resonance models [8].

DISCUSSION

Given the experimental validity of proposition 1, we have thus exhibited a clash between scaling and the simplest generalization of the EVMD hypothesis to multiple-photon processes. To find a way out, one might, for example, argue that although unsubracted dispersion relations seem to work well for one-photon processes, when going to multiple photon processes, one should go to subtracted dispersion relations (SDR).

With a once-SDR for example, one finds instead of Eq. (8) a series that is a power of $\frac{1}{m_n^2}$ more convergent. Howeve, because of the subtraction constant, the model can provide non-trivial information on $\frac{d}{d q^2} T$ but not on $T(p^2, 0, 0)$. This is equivalent to saying that the model has nothing to say about the absolute $\pi^0 \to 2\gamma$ decay rate. The EVMD model would then be a rather restricted model providing information on mass extrapolations but not on absolute rates for multi-photon processes.

Presumably, we would then be better off using only a small number of vector mesons to saturate the low mass region and having the high mass region described in some other way, i. e., one might as well abandon the hope of a complete description of hadronic e. m. interactions by a model with infinitely many poles and suggest that a FMDR approach seemed more reasonable.

In the above discussion, I have analyzed the implications of scaling and $\pi^0 \to 2\gamma$ for the prospects of a complete EVMD description of photon-hadron interactions. If, on the other hand, one takes the unpopular view that the storage rings results do not yet prove asymptotic scaling in $e^+ e^-$ and if it turns out that scale is broken after all (for instance at distances of order 10^{-15} cm. as it has been suggested [9]), then the present result leads to the intriguing suggestion that such breaking might somehow be related to the finiteness of the $\pi^0 \to 2\gamma$ width, i. e., to the very existence of an observable neutral pion.

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