ON THE DIVERGENCE OF ENTHALPY AND THE ENERGETICS OF THE ATMOSPHERE (*)

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ABSTRACT — The role of the divergence of enthalpy in the energetics of the atmosphere is studied and discussed. The heating due to the divergence or the convergence of enthalpy can be regarded in the study of the energetics of the atmosphere as a basic tool in evaluating its energy budget. The analysis of the energy equation applied to the atmosphere shows that the contribution of the term of enthalpy becomes dominant. A balance equation for the enthalpy is established and the physical interpretation and the relative contribution of its various terms are fully discussed. Since the divergence or the convergence of the enthalpy flux reflect the excess or the deficit of diabatic heating, as follows from the balance equation, the present approach offers an independent way of checking the global energy budget of the Earth and of the atmosphere. The divergence of enthalpy acting in an opposite sense to the adiabative heating offers through its spacial distribution a map of the heat sources and sinks of the system earth-atmosphere, which may be of importance for the mathematical simulation of the general circulation.

1 — INTRODUCTION

The distribution of the diabatic effects in the earth-atmosphere system is essential for the understanding of the mechanisms of the physical processes which produce and maintain the various scales

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of motion of the atmosphere and lead to its general circulation (Smagorinsky, 1953).

The general circulation of the atmosphere can be regarded as a generalized convective phenomenon in which the possible regimes of motion depend on the meridional gradient of the temperature of the air between the equator and the poles, on the rate of rotation of the earth and on the nature of the boundary conditions. The nonuniform heating of the atmosphere in the tropical and polar regions and the inhomogeneity of the earth lead to density differences which upset the stable barotropic stratification of the atmosphere and generate *total potential energy* (potential plus internal energy) which is partially *available* for conversion into *kinetic energy* of the motion (Lorenz, 1967).

In a nonrotating Earth, the existing meridional gradient of temperature would produce a direct toroidal circulation, axially symmetric, between the equator and the poles (Hadley regime). However, the Coriolis deflecting force due to the Earth's rotation is strong enough to deform and prevent the purely celular-toroidal regimes of the general circulation of the atmosphere. Under these circumstances, it would seem that the transport of enthalpy, from the regions in which a permanent increase prevails (tropical regions) to those in which a permanent decrease exists (polar and sub-polar regions) would cease and this situation would lead to a steady increase of the latitudinal gradient of temperature. However, beyond a certain critical value of the latitudinal temperature gradient the atmosphere becomes baroclinically unstable, with the formation and selective development of eddies which deform the initially symmetric fields of motion and temperature. Once the dynamic stability is again attained the so called Rossby regime is established. In this regime characterized by large horizontal perturbations, the eddies carry on the excess of enthalpy, accumulated in tropical regions, in a quasi-horizontal macro turbulent processes and set up, simultaneously, the balance of angular momentum, which mantains the westerlies in middle latitudes and the easterlies in tropical regions (Lettau, 1954). The final adjustment of these balances in the Rossby regime can lead to the establishment of a secondary toroidal regime, formed by a tricelular residual circulation with two direct cells one over the equatorial region and another over the polar regions, the Hadley cells, and an indirect one the so called Ferrel cell (Lorenz, 1967).

The continuous dissipation of the kinetic energy of the organized atmospheric motions, due to friction and to turbulent and molecular viscosity, would in the limit lead the atmosphere, to a state of rest relative to the Earth. As a consequence, the atmosphere would rotate rigidly with the Earth, and no differential rotation could exist. This situation is, of course, against the observed winds and ocean currents, which show the existence of a differentiated and nonstationary character of the atmospheric and oceanographic motions. This implies the existence in the atmosphere of transformations of other forms of energy which will lead to the production of kinetic energy and allow its regeneration against all dissipative processes and guarantee the maintenance of the general circulation. We can thus say that the fundamental problem of atmospheric energetics is the determination of the processes that convert the radiant energy from the sun, which is the primary source of all atmospheric energy, into the kinetic energy of the planetary circulations. The conversion processes of the various forms of energy are not random and the various mechanisms show that otherwise a certain order exists in the sequence of the transformations of the various forms of energy, leading to the concept of an energy cycle in the atmosphere (Peixoto, 1965; Starr, 1968).

A simplified version of this cycle is as follows: the absorption of solar radiation and emission of terrestrial radiation set up a meridional (or latitudinal) gradient of temperature as a result of the non equal heating of the atmosphere in the tropical and polar regions. From a quasi-zonal temperature distribution resulting from the almost axially-symmetric heating, zonal available potential energy is established. Later the distortion of the ideally axially-symmetric fields of motion and of the enthalpy and latent heat leads to a partial conversion of this form of energy into eddy available potential energy, through the meridional transport of enthalpy accomplished by the atmospheric circulations. This latter form of energy is, in turn, converted by baroclinic processes into kinetic energy of the eddies, with ascention of warmer air and the descent of colder air (White and Saltzman, 1956). Finally, by barotropic processes, the eddies feed part of their energy to the mean zonal current (Starr and Wallace, 1964; Starr, 1968). The zonal kinetic energy is mostly dissipated by friction, turbulence and viscosity in a cascade process (Kolmogoroff) into heat. The remainder of this form of energy is converted into total available potential energy by the mean indirect meridional circulations (Ferrel cell). The dissipated energy into heat is finally reradiated to the space as infrared radiation, and thus, closing the cycle.

This is, of course, a very simplied version of the scheme. Actually there are other forms of energy conversion in the atmosphere which have to be accounted for in the energy cycle such as those related to the energy released in the transitions of phase of the water vapour (Peixoto, 1965) and those involved in the absorption and emission of radiant energy, etc.

Besides the global aspects of the energetics of the atmosphere in the study of the mechanisms responsible for the maintenance of the general circulation, the geographic-synoptic distribution of the diabatic effects which originate an nonuniform heating of the atmosphere and consequently generate total available potential energy assumes a fundamental importance (Davis, 1963). However, the field of the distribution of diabatic effects can be properly expressed by the field of the divergence of enthalpy in the atmosphere, as we will discuss in the present paper.

In the discussion that follows we will present a theoretical formulation of the general problem of energetics of the atmosphere in the spirit of our previous work, where the importance of the enthalpy fields has been stressed (Peixoto, 1974). However some concepts will be treated with more detail, as required by the present study.

2 — FORMULATION OF THE PROBLEM

2. 1. *Balance equation of the energy*

In the study of the energetics of the atmosphere all the energy forms play a role, but we will restrict ourselves to those which are predominant for the actual study, namely the mechanical, thermal and radiant forms. Under these conditions, the atmosphere can be regarded as an open and nonisolated thermomechanical system. Other forms of energy, such as electric, chemical, nuclear, etc., can be disregarded because their contribution is very small when compared to the forms of radiant, internal, kinetic and potential energies.

The fundamental equation of the energetics of the system earth-atmosphere is obtained by combining the equation of mechanical energy resulting from the equation of motion and the equation of continuity with the first law of thermodynamics (Peixoto, 1965).

Under these conditions the general energy equation of the system earth-atmosphere can be written in the form of a balance equation for the unit volume as follows (Peixoto, 1967)

$$\frac{\partial}{\partial t}\rho(\mathrm{U}+\mathrm{K}+\Phi)+\operatorname{div}\rho(c_{\mathrm{V}}\mathrm{T}+p\alpha+\mathrm{L}q+\Phi+\mathrm{K})\vec{\mathrm{V}}-$$

$$-\rho\vec{\mathrm{F}}\cdot\vec{\mathrm{V}}=\rho\frac{d\mathrm{Q}}{dt} \qquad (1)$$

where $\frac{\partial}{\partial t}\rho(\mathrm{U}+\mathrm{K}+\Phi)=\frac{\partial \mathrm{E}}{\partial t}$ represents the local rate of change of the total energy E, ρ the density, U the internal energy, K the kinetic energy; in the divergence term p denotes the atmospheric pressure, α the specific volume of air, L the latent heat of condensation, q the specific humidity of air, $\vec{\mathrm{V}}$ the vector field of the wind velocity and $\vec{\mathrm{F}}$ the specific frictional force due to small-scale turbulence and the stresses at the boundary.

Finally $\rho\frac{d\mathrm{Q}}{dt}$ is the rate of heating or cooling in the atmosphere due to diabatic effects, namely to conduction and friction $\rho\frac{d\mathrm{Q_F}}{dt}$, to radiation $\rho\frac{d\mathrm{Q_R}}{dt}$ and to condensation $\rho\frac{d\mathrm{Q_L}}{dt}$.

Equation (1) has the general form of a balance equation: the local rate of change of energy is associated to a field of energy current density which are the response to the existence of sources or sinks, represented by the dissipation term and by the second member; this, in fact, gives the total rate of production or destruction of energy. The local rate of change of energy is due to:

a) The total flux, represented by the energy current density vector $\vec{\mathrm{J}}_{\mathrm{E}}$ which appears in the argument of the divergence term:

$$\vec{\mathrm{J}}_{\mathrm{E}}=\rho(c_{\mathrm{V}}\mathrm{T}+p\alpha+\mathrm{L}q+\Phi+\mathrm{K})\vec{\mathrm{V}}$$

b) the dissipation of the energy of motion due to friction forces represented by the term: $\rho\vec{\mathrm{F}}\cdot\vec{\mathrm{V}}$, which can produce important tangential stresses, such as those generating ocean currents, etc.

c) The production or destruction of energy by diabatic effects, represented by $\rho\frac{d\mathrm{Q}}{dt}$.

In the balance equation the divergence term represents the interaction of the unit volume particle with the surroundings. On the other hand if a physical quantity is an invariant, its rate of production or destruction is zero; therefore any local variation of the quantity due to the interaction with the surroundings is given by the divergence term. If, on the contrary, the particle constitutes a closed system for a given property, its local variation is only due to an internal production or destruction of the property within the particle.

For long periods of time the energy storage rate of change in the atmosphere is probably small and the balance equation (1) reduces to the form:

$$\operatorname{div} \rho (c_p \mathrm{T} + \mathrm{L} q + \Phi + \mathrm{K}) \vec{\mathrm{V}} - \rho \vec{\mathrm{F}} \cdot \vec{\mathrm{V}} = \rho \frac{d \mathrm{Q}}{d t} . \tag{2}$$

The kinetic energy of existing motions is at least two orders of magnitude smaller than the other forms of energy and can be disregarded in equations (1) and (2). The enthalpy and the potential energy are, together with latent heat, the major components of the energy transported by the atmosphere to fulfill the balance requirements of the general circulation of the atmosphere. In view of the importance of the enthalpy in the atmosphere in the discussion that follows the balance of the enthalpy is discussed with some detail.

2. 2. *The enthalpy balance equation*

Let $h = c_p \mathrm{T}$ be the specific enthalpy of air, considered a perfect gas, at a given point of the atmosphere where the temperature is T at the instant t. The corresponding value of the absolute enthalpy will be ρh. Under these conditions the general balance equation of enthalpy in a generalized local frame of reference $(0, x^i)$ can be written in the form:

$$\frac{\partial}{\partial t} \rho h + \frac{1}{\sqrt{\mathrm{G}}} \frac{\partial}{\partial x^i} [\sqrt{\mathrm{G}} \rho h v^i] = \rho \frac{d h}{d t} = \sigma(h) \tag{3}$$

where $\mathrm{G} \equiv |g_{ij}|$ is the determinant of the tensor of the metric, v_i is the contravariant component of the air velocity at the point x^i, at a given instant t; $\sigma(h)$ denotes the time rate of production or destruction of enthalpy per unit volume.

Equation (3) shows that the local variation of enthalpy results from the divergence of the transport of enthalpy, and from the production or destruction of enthalpy, $\sigma(h)$.

Let us consider, at a point of the surface of the earth a column of air of unit section, extending from the ground to the top of atmosphere. The balance equation (3) can be integrated along the vertical. The resulting equation is:

$$\frac{\partial}{\partial t}\int_0^\infty \rho\, h\, dz + \frac{1}{\sqrt{G}}\,\frac{\partial}{\partial x^i}\int_0^\infty [\sqrt{G}\,\rho\, h\, v^i]\, dz =$$

$$= \int_0^\infty \rho \frac{dh}{dt}\, dz = \Sigma_t(h)\,. \tag{4}$$

Assuming that the atmosphere is in hydrostatic equilibrium, $dp = -\rho g\, dz$ where g is the acceleration of the gravity, this equation can be rewritten in a p-system taking the corresponding boundary conditions ($p = p_0$, when $z = 0$ and $p = 0$ for $z = \infty$).

Let us denote by the total enthalpy of the air column which in the p-system is then given by

$$H = \frac{c_p}{g}\int_0^{p_0} T\, dp\,. \tag{5}$$

Similarly let us introduce the contravariant component S^i of the total integrated field of transport of enthalpy. In the p-system is then defined by

$$S^i = \frac{c_p}{g}\int_0^{p_0} T\, v^i\, dp\,. \tag{6}$$

With these notations equation (4) in the p-system takes the form:

$$\frac{\partial H}{\partial t} + \frac{1}{\sqrt{G}}\,\frac{\partial}{\partial x^\alpha}\sqrt{G}\, S^\alpha = \frac{1}{g}\int_0^{p_0} \sigma(h)\, dp = \Sigma_t(h) \tag{7}$$

in which $\Sigma_t(h)$ represents the total rate of production or destruction of enthalpy within the column of unit section.

Because of the earth's spherical symmetry it is convenient to take as the reference frame $(0, x^i)$ a system of quasi-lagrangean spherical coordinates (λ, φ, p, t) where λ is the longitude, φ the latitude, p is taken as vertical coordinate and t the time. In this coordinate system the integrated balance equation can be written as follows:

$$\frac{\partial H}{\partial t} + \frac{1}{a^2 \cos\varphi}\left[\frac{\partial}{\partial\lambda}(S_\lambda a) + \frac{\partial}{\partial\varphi}(S_\varphi a \cos\varphi) + \right.$$

$$\left. + \frac{\partial}{\partial p}(S_p a^2 \cos\varphi)\right] = \Sigma_t(h). \tag{8}$$

In this equation a is the radius of the earth, S_λ, S_φ and S_p are the physical components of the zonal, meridional and vertical transports of enthalpy vertically integrated above a point at the earth's surface given by

$$\begin{aligned} S_\lambda &= \frac{c_p}{g}\int_0^{p_0} u\,T\,dp \\ S_\varphi &= \frac{c_p}{g}\int_0^{p_0} v\,T\,dp \\ S_p &= \frac{c_p}{g}\int_0^{p_0} \omega\,T\,dp \end{aligned} \tag{9}$$

noting that the wind field $\vec{V}$ is given by

$$\vec{V} = u\vec{i} + v\vec{j} + \omega\vec{k} \tag{10}$$

where u is the zonal component counted positively to the east, v the meridional component taken positive to the north and $\omega = \frac{dp}{dt}$ is the «vertical component» of the wind in the p-system.

The rate of enthalpy generation $\Sigma_t(h)$ due to atmospheric sources or sinks, is given by the term of diabatic effects $\rho\frac{dQ}{dt} = \rho\dot{Q}$ as will be shown. In the p-system the rate of heating or cooling $\frac{dQ}{dt}$ is, according to the first law of thermodynamics, given by

$$\frac{dQ}{dt} = \frac{dh}{dt} - \alpha\frac{dp}{dt} \tag{11}$$

where $\alpha = \frac{1}{\rho}$.

Expanding the term $\frac{dh}{dt}$ and using the continuity equation in the p-system, this equation, noting that $\omega = \frac{dp}{dt}$, becomes

$$\frac{dQ}{dt} = \frac{\partial h}{\partial t} + \mathrm{div}_p\, h \vec{V} + \frac{\partial}{\partial p}(h\omega) - \alpha\omega . \qquad (12)$$

The first term of this equation is as we have mentioned the rate of production due to diabatic effects. In the second member, the term $\frac{\partial h}{\partial t}$ gives the local variation of enthalpy, in the atmosphere; the terms $\mathrm{div}_p\, h \vec{V}$ and $\frac{\partial}{\partial p}(h\omega)$ are the divergence components of enthalpy over and across isobaric surfaces, respectively of the horizontal and of the vertical fluxes. The term $\alpha\omega = \frac{RT}{p}\omega$ represents, as it is well known, the rate of conversion of the total potential energy (internal plus potential) into kinetic energy (Saltzman and White, 1956). This rate of conversion is small compared to the other terms of the R. H. S. Thus we can conclude that the diabatic effects $\frac{dQ}{dt} \equiv \dot{Q}$ constitute practically the rate of production or destruction of the enthalpy in the atmosphere. Or, yet, the field of the divergence of enthalpy constitutes a good representation of the geographical distribution of the heat sources ($\mathrm{div} > 0$) and of the heat sinks ($\mathrm{div} < 0$) for the atmosphere. It gives the spacial distribution of the globe diabatic effects which condition the behaviour of the general circulation of the atmosphere.

3 — ANALYSIS OF THE BALANCE EQUATION OF ENTHALPY

Among the various diabatic effects responsible for the production of enthalpy in a certain region we can enhance the effects due to the absorption of solar (short wave lenghts) and terrestrial radiation (long wave lenghts) jointly represented by $\dot{Q}_R$; the effects due to the conduction of sensible heat from the ground $\dot{Q}_C$, the dissipation due to friction $\dot{Q}_F$ and the effect associated with phase transitions (condensation or evaporation) of water $\dot{Q}_L$. Therefore for the total diabatic effects the time rate of change is (Houghton, 1954; London, 1957)

$$\dot{Q} = \dot{Q}_R + \dot{Q}_C + \dot{Q}_F + \dot{Q}_L . \qquad (13)$$

Therefore the enthalpy divergence gives the total rate of change of the diabatic effects.

The preceding enthalpy equation becomes more suggestive if it is expressed in terms of temperature and of the transport field of enthalpy $h\vec{V}$. Therefore we can write:

$$\frac{dQ}{dt} = \frac{\partial h}{\partial t} + \operatorname{div} h\vec{V} + C_p \frac{\partial(\omega T)}{\partial p} - \alpha\omega \qquad (14)$$

or

$$\frac{\partial T}{\partial t} = \frac{1}{c_p}\frac{dQ}{dt} - \operatorname{div} T\vec{V} - \frac{\partial(T\omega)}{\partial p} + \alpha\omega / c_p . \qquad (14a)$$

This equation shows, immediately, that the local variation of temperature $\frac{\partial T}{\partial t}$ is mainly due to the diabatic effects $\frac{dQ}{dt}$ and to the field of divergence of enthalpy, since the other terms give only a very small contribution.

To stress the importance of various diabatic effects we can still put that equation in the form:

$$\dot{Q}_R + \dot{Q}_F + \dot{Q}_C + \dot{Q}_L = c_p \frac{\partial T}{\partial t} + c_p \operatorname{div} T\vec{V} + c_p \frac{\partial(T\omega)}{\partial p} - \alpha\omega . \qquad (15)$$

Integrating this equation between the levels p and $p - \Delta p$, and noting that $dp = -\rho g\, dz$, it comes, using the operator $\{\,\}$ to represent the vertical integration

$$\frac{\Delta p}{g} \Sigma \{\dot{Q}_i\} = \frac{c_p}{g} \Delta p \left\{\frac{\partial T}{\partial t}\right\} + c_p \frac{\Delta p}{g} \{\operatorname{div} T\vec{V}\} +$$

$$+ \frac{1}{g}\{c_p T\omega\}_p - \frac{1}{g}\{c_p T\omega\}_{p-\Delta p} - \frac{\Delta p}{g}\{\alpha\omega\} \qquad (16)$$

as it is readily seen by using the theorem of mean value of integral calculus.

This equation shows that the vertical transport of enthalpy $\frac{1}{g}\left\{\frac{\partial}{\partial p} c_p T \omega\right\}$ given by the difference

$$\frac{1}{g}(c_p T \omega)_p - \frac{1}{g}(c_p T \omega)_{p-\Delta p} \tag{17}$$

is very small because the values of ω are very small and the temperature at the two levels are almost equal.

Particularly if the whole atmosphere is taken into account, that is, when the integration is performed between the levels p_0 and $p=0$ that equation can be written in a simple form, since $\omega(0)=0$ and $\omega(p_0)=0$ through the interface earth-atmosphere; under these conditions the equations (14) and (14a) respectively take the forms:

$$\frac{c_p}{g} p_0 \left\{\frac{\partial T}{\partial t}\right\} + \operatorname{div} \vec{S} = \Sigma\, \dot{Q}_i \tag{18}$$

and

$$\frac{\partial H}{\partial t} + \operatorname{div} \vec{S} = \Sigma \{\dot{Q}_i\}\,. \tag{19}$$

Equation (19) reveals that the local variation of enthalpy at a given point, represented by the first member, is due to the flux of enthalpy, given by the divergence term and to the production or destruction of enthalpy *in situ* given by the second member.

This equation which is none but a new form of the balance equation, can be written explicitly in the frame (λ, φ, p, t) as follows:

$$\frac{\partial H}{\partial t} + \frac{1}{a\cos\varphi}\left\{\frac{\partial}{\partial\lambda} S_\lambda + \frac{\partial}{\partial\varphi}(S_\varphi \cos\varphi)\right\} = \Sigma\{\dot{Q}_i\}\,. \tag{20}$$

For a region of the atmosphere bounded by the earth's surface of area A limited by the contour (c) this equation can be written in the form:

$$\frac{\partial H}{\partial t} + \frac{1}{A}\oint_c (\vec{S}\cdot\vec{n})\, dc = \Sigma\{\dot{Q}_i\} \tag{21}$$

where $\vec{n}$ is the normal vector to contour (c).

The application of the preceding equation to regions bounded by latitudinal walls, $\Delta \varphi$ degrees apart, leads to a new and more simplified form of the balance equation, as results from the application of the Ostrogradsky-Gauss theorem. It comes:

$$\frac{\partial H}{\partial t} + \frac{1}{a \cos \varphi} \frac{\partial}{\partial \varphi} \oint S_\varphi (\lambda, \varphi) \cos \varphi \, d\lambda = \Sigma \{ \dot{Q}_i \} \qquad (22)$$

since

$$\oint \frac{\partial S_\lambda}{\partial \lambda} d\lambda = 0 \, . \qquad (23)$$

In equation (22) only the meridional convergence of enthalpy across the latitudinal walls φ and $\varphi + \Delta \varphi$ appears.

4 — MODES OF DIVERGENCE OF ENTHALPY IN THE ATMOSPHERE

In the atmosphere one can distinguish various types of circulations. When one considers the mean circulations in the space and in time domains the motions can be partitioned in two general categories, namely the mean and the eddy circulations.

Thus the eastward, the northward and the vertical components of the wind u, v and ω and the temperature T may be expressed as the sum of four components

$$\begin{aligned} u &= [\bar{u}] + \bar{u}^* + [u]' + u'^* \\ v &= [\bar{v}] + \bar{v}^* + [v]' + v'^* \\ \omega &= [\bar{\omega}] + \bar{\omega}^* + [\omega]' + \omega'^* \\ T &= [\bar{T}] + \bar{T}^* + [T]' + T'^* \end{aligned} \qquad (24)$$

where the bar represents a time average and the prime a deviation from the time average, the brackets a zonal average, the asterisk a deviation from the zonal average.

In the time domain the mean of the products which appear in expressions (9) may be expanded according to the scheme

$$\overline{\mathrm{T}u} = \overline{\mathrm{T}}\,\overline{u} + \overline{\mathrm{T}'u'}$$
$$\overline{\mathrm{T}v} = \overline{\mathrm{T}}\,\overline{v} + \overline{\mathrm{T}'v'} \qquad (25)$$
$$\overline{\mathrm{T}\omega} = \overline{\mathrm{T}}\,\overline{\omega} + \overline{\mathrm{T}'\omega'}.$$

Averaging zonally around a latitude circle a resolution of these expressions in the mixed space-time domain is obtained using the «bracket-operator»:

$$[\overline{\mathrm{T}u}] = [\overline{\mathrm{T}}][\overline{u}] + [\overline{\mathrm{T}}^*\,\overline{u}^*] + [\overline{\mathrm{T}'u'}]$$
$$[\overline{\mathrm{T}v}] = [\overline{\mathrm{T}}][\overline{v}] + [\overline{\mathrm{T}}^*\,\overline{v}^*] + [\overline{\mathrm{T}'v'}] \qquad (26)$$
$$[\overline{\mathrm{T}\omega}] = [\overline{\mathrm{T}}][\overline{\omega}] + [\overline{\mathrm{T}}^*\,\overline{\omega}^*] + [\overline{\mathrm{T}'\omega'}].$$

Different schemes for the expansions of the mean total transfer of enthalpy could be used (Starr and White, 1954; Peixoto 1960). However, the type of expansion (26) is the most useful in dealing with general circulation problems. The interpretation of the various terms of expansions (26) is very instructive, since each one corresponds to a mode of enthalpy flux (Peixoto, 1960). In fact the terms $\frac{c_p}{g}[\overline{\mathrm{T}}][\overline{u}]$ and $\frac{c_p}{g}[\overline{\mathrm{T}}][\overline{v}]$ represent the transport of enthalpy at a given isobaric level by the zonally averaged zonal and meridional wind fields. Particularly the term $\frac{c_p}{g}[\overline{\mathrm{T}}][\overline{v}]$ measures the contribution for the total meridional flow of enthalpy of the mean meridional cells. The terms $\frac{c_p}{g}[\overline{\mathrm{T}}^*\,\overline{u}^*]$ and $\frac{c_p}{g}[\overline{\mathrm{T}}^*\,\overline{v}^*]$ represent the transfer due to spatial covariance between the mean time averaged field departures from zonal symmetry and are associated with the mean standing eddies. Finally the terms $\frac{c_p}{g}[\overline{\mathrm{T}'u'}]$ and $\frac{c_p}{g}[\overline{\mathrm{T}'v'}]$ arise from the zonally averaged covariances of the instantaneous local values, and represent the local zonal and meridional mean transfers of enthalpy associated with the transient horizontal eddies (Starr and Wallace, 1964). Similar interpretations apply to the various terms of the total mean vertical transports; in particular the term $\frac{c_p}{g}[\overline{\mathrm{T}}][\overline{\omega}]$ corresponds to the vertical transport of enthalpy by the ascending and descending branches of the meridional cells, etc.

The integration along the vertical of equations (26) multiplied by C_p/g leads to the corresponding terms of the various modes of transport of enthalpy for all the atmosphere

$$[\vec{\overline{S}}] = \vec{S}_M + \vec{S}^* + \vec{S}' \qquad (27)$$

whose components are:

$$\begin{aligned} [\overline{S}_\lambda] &= S_{\lambda M} + S^*_\lambda + S'_\lambda \\ [\overline{S}_\varphi] &= S_{\varphi M} + S^*_\varphi + S'_\varphi \\ [\overline{S}_p] &= S_{p M} + S^*_p + S'_p . \end{aligned} \qquad (28)$$

Thus

$$\operatorname{div} [\vec{\overline{S}}] = \operatorname{div} \vec{S}_M + \operatorname{div} \vec{S}^* + \operatorname{div} \vec{S}' \qquad (29)$$

and the divergence of enthalpy on a global scale is due to the divergence accomplished by the mean meridional circulations, by the standing eddies and by the transient eddies.

The eddies are in general predominantly horizontal, with a meridional and a zonal component, or vertical. The meridional circulations are formed by the so called meridional cells. Due to the uncertainty involved in the evaluation of $[\overline{v}]$ (for the geostrophic approximation it is identically zero), the estimate of the mean meridional transport of enthalpy by the mean meridional circulations is difficult to assess. As the $[\overline{v}]$ values can be significant in the lower levels of the equatorial Hadley cell, the divergence (or convergence) of enthalpy due to this mechanism can be important; the same happens with the divergence or the convergence associated with the vertical branches of the Hadley circulation (Newell, *et al*, 1970).

5 — FINAL COMMENTS

1. As the atmosphere ability for storing enthalpy leads only to small variations we can assume, with a negligible error, that for a sufficiently long interval of time, such as a season, the corresponding local variation of the time mean of enthalpy may be set to zero

$$\frac{\partial \overline{H}}{\partial t} = 0 . \qquad (30)$$

Therefore, it can be concluded that, on the average, the field of the mean divergence of total enthalpy transport in the atmosphere for any region is balanced by the diabatic effects as results from the time averaged equation (19). Therefore, through the computation of the mean divergence of enthalpy, a dynamic method of verifying the consistency of estimates of the energetic balance of earth and of atmosphere obtained by traditional approach is provided.

2. When we consider a zonal ring of the atmosphere bounded by the latitudes φ and $\varphi + \Delta\varphi$ the mean divergence is reduced to mean meridional components, since

$$\left[\frac{\partial \overline{S}_\lambda}{\partial \lambda}\right] = \frac{1}{2\pi}\int_0^{2\pi} \frac{\partial \overline{S}_\lambda}{\partial \lambda}\, d\lambda = 0\,. \tag{31}$$

Under steady state conditions and for a zonal region of the atmosphere, taking into consideration (28), equation (20) reduces to the form

$$\frac{1}{a\cos\varphi}\,\frac{\partial}{\partial\varphi}\left\{([\overline{S}_{\varphi M}] + S^*_\varphi + S'_\varphi)\cos\varphi\right\} = \Sigma\,\{\dot{Q}_i\}\,. \tag{32}$$

Therefore, from the derivatives of the profiles of the various modes of meridional flux of enthalpy, one can obtain, in principle, the mean zonal distribution of diabatic effects for the system earth--atmosphere.

3. Let us analyse the situation level by level. One method of examining the consistency of the studies of the atmospheric heat balance at a given level is to consider the various factors which control the mean temperature distribution. In a given region, temperature changes are due to diabatic heating or cooling and to heat transport by the various modes of atmospheric motions. These are more explicitly revealed when equation (14a) is averaged zonally and with respect to time. In fact it can be written *in extenso* in spherical coordinates taking into consideration the expressions (26), as follows:

$$\frac{\partial[\overline{T}]}{\partial t} = \frac{1}{c_p}\left[\overline{\frac{dQ}{dt}}\right] + \gamma[\overline{\omega}] - \frac{1}{a\cos\varphi}\,\frac{\partial}{\partial\varphi}[\overline{V'T'} + \overline{V}^*\,\overline{T}^*]\cos\varphi$$
$$-\frac{[\overline{V}]}{a}\,\frac{\partial[\overline{T})}{\partial\varphi} - \frac{\partial}{\partial p}[\overline{\omega' T'} + \overline{\omega}^*\,\overline{T}^*] + \frac{R}{p\,c_p}[\overline{\omega' T'} + \overline{\omega}^*\,\overline{T}^*] \tag{33}$$

where γ is the static stability factor, given by

$$\gamma = \frac{R}{p\,c_p}[\overline{T}] - \frac{\partial[\overline{T}]}{\partial p}. \qquad (34)$$

The terms on the right hand side (R. H. S.) of Eq. (33) can be inferred from the observations with the exception of the vertical heat fluxes.

Let us analyse this equation in detail. The magnitudes of the first three terms on the R. H. S. are of the order of one degree per day, whereas the fourth term, only important in the lower boundary layer, is of the order of 10^{-1} degrees per day. Estimates of the fifth and sixth terms using vertical heat fluxes derived from an 18-level general circulation model were also of the order of 10^{-1} degrees per day (Manabe and Hunt, 1968), although the fifth in the upper troposphere over the middle latitude regions can, at times, approach values of near one degree per day. Since typical values of the time derivative are of the order of 10^{-1} degrees per day the year and the seasonal heat balances can be treated as steady state problems. Thus, as the contributions by the last terms are insignificant, the zonal balance at a given level is governed, to a first approximation, by the three first terms, namely diabatic heating or cooling, adiabatic compression or expansion and meridional eddy convergence (divergence) of enthalpy.

4. In view of the comments already made, and since the heating due to the divergence of enthalpy acts in a sense opposite to the radiative heating, in principle, a map of the mean total divergence of enthalpy, $\operatorname{div}[\overline{\vec{S}}]$, can be regarded as a map of the energy budget of the system earth-atmosphere. Due to the importance that the parameterization of the diabatic effects assumes in the mathematical modelling of the general circulation, such a map can be considered as a first approximation to define the distribution of the «heating function». With this approach it would then be possible from observed quantities to obtain a spacial representation of the heat sources and sinks, that is to say the corresponding distribution of diabatic effects.

Furthermore, the time average physical state of the atmosphere can be regarded as a forced response to the geographically fixed distribution of the mean diabatic effects, topography and to the mean convergence fields of enthalpy, latent heat and momentum associated to the transient eddies. In particular the forcing F_S due to the enthalpy flux is given by the vertical derivative of the eddy flux of enthalpy

$$F_S = - f \frac{\partial}{\partial p} \left(\frac{1}{\gamma} \operatorname{div} \vec{S}' \right) \tag{35}$$

where $f = 2\omega \operatorname{sen} \varphi$ denotes the Coriolis parameter and γ the static stability. This expression shows how important the bidimensional fields of the eddy transport of enthalpy at various isobaric levels are in studying the dynamics of the general circulation of the atmosphere.

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