TRANSFER REACTIONS WITH HEAVY-IONS (*)

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ABSTRACT — An extension of the Buttle and Goldfarb formalism for heavy ion transfer reactions is presented where recoil effects are considered using the local momentum approximation and the transition amplitude retains the structure of the zero range approximation. The reliability of the method was tested with numerical calculations which take a very small computing time compared with full finite range calculations.

It is well known that transfer reactions induced by heavy ions are a valuable tool in nuclear spectroscopy, in particular to study high spin nuclear states. The distorted wave Born approximation (DWBA), which is the basic theory for a quantitative quantal treatment of the reaction mechanism, is frequently used assuming the no-recoil approximation since a full finite range calculation requires large computer times. A DWBA code was developed where the corrections due to the recoil of the heavy ion cores are introduced in an approximate way performing a Taylor expansion of the distorted waves. The non-normal parity orbital angular momentum transfer is then automatically included and the terms in the expansion of the reduced amplitude can be characterized by the recoil orbital angular momentum transfer l_r. Due to the strong localization of the transfer it is usually a good approximation to represent the bound state wave function by the asymptotic Hankel function. With this description the transition amplitude can be calculated exactly using a local momentum approximation.

In the DWBA transition amplitude for the transfer reaction A (a, b) B, where $a = b + x$, and x is the transferred particle, a Taylor

(*) Sponsored in part by Centro de Física Nucler (INIC); communication delivered at the Conference of the Portuguese Physics Society (Lisbon, February 1978).

expansion of the distorted waves is performed at the displacement vector of the two ion cores. Using an irreducible tensor expansion of the translation operator, we obtain for each value of the transferred angular momentum l, a series of terms, characterized by an orbital angular momentum l_r that satisfies the vector relation

$$\vec{l}_r = \vec{l} - \vec{l}' , \tag{1}$$

where $\vec{l}'$ is the angular momentum transferred from the relative motion of the ion cores in the entrance channel to the relative motion of the ion cores in the exit channel. We shall call $\vec{l}_r$ the recoil angular momentum. This variable is associated with linear recoil momentum

$$\vec{p} = -\left(\frac{M_x}{M_B} \vec{q}_b + \frac{M_x}{M_a} \vec{q}_a \right) \tag{2}$$

where M_i is the mass of the particle i, and $\vec{q}_a$ ($\vec{q}_b$) is the relative momentum in the entrance (exit) channel [1].

The above angular momentum quantum numbers satisfy the parity rule [2]

$$l' + l_r + L + L' = \text{even} \tag{3}$$

where L (L′) is the orbital angular momentum of x in the projectile (residual nucleus). The no-recoil approximation is obtained when we take into account only the no-recoil term of the series expansion of the transition amplitude corresponding to $l_r = 0$, which implies $l = l'$. In this approximation the non-normal parity values of l (without the parity of $L + L'$) do not contribute to the cross section.

Following Braun-Munzinger and Harney [3], we assume that in the asymptotic region where the transfer is more probable, the following approximation in the recoil terms can be made

$$\left(\frac{M_x}{M_A} \nabla_b - \frac{M_x}{M_a} \nabla_a \right) \chi^{(-)*}_{\vec{k}_b} (\gamma \vec{R}) \, \chi^{(+)}_{\vec{k}_a} (\vec{R}) =$$

$$i \, \vec{p} \, \chi^{(-)*}_{\vec{k}_b} (\gamma \vec{R}) \, \chi^{(+)}_{\vec{k}_a} (\vec{R}) \tag{4}$$

where $\gamma = M_A / M_B$, $\vec{R}$ is the displacement vector between the ion cores, $\chi^{(+)}_{\vec{k}_a}$ and $\chi^{(-)}_{\vec{k}_b}$ are the elastic scattering wave functions which describe, respectively, the relative motion of the pair A, a before the transfer, and the pair B, b after the transfer, and ∇_b (∇_a) is the gradient operator with respect to $\vec{R}$ and acts only on $\chi^{(-)}_{\vec{k}_b}$ ($\chi^{(+)}_{\vec{k}_a}$). In eq. (2) $\vec{q}_b$ and $\vec{q}_a$ are the local relative momenta in the exit and entrance channels. The modulus of $\vec{q}$ is determined by the relation

$$q\,(r_o) = \left[\frac{2\,\mu}{\hbar^2}\,(\mathrm{E} - \mathrm{U}_{\mathrm{opt}}\,(r_o))\right]^{1/2} \tag{5}$$

where μ is the reduced mass, E is the energy in the CM system, r_o is the interaction radius and U_{opt} is the optical potential that generates χ. The direction of $\vec{q}$ is assumed to be perpendicular to $\vec{R}$. This means that as regards the treatment of the recoil terms we assume that the transfer is more probable in the direction of $\vec{R}$.

The grazing collision character of heavy-ion reactions suggests that at least below the Coulomb barrier, where the no-recoil approximation is reasonable, the bound state wave functions may be approximated by the spherical Hankel function which describes their asymptotic behaviour [4].

With these assumptions, the reduced amplitude has the very simple form

$$\beta^{\mathrm{LL}'l\lambda}_{sj} = \frac{4\,\pi}{\sqrt{2\,l+1}}\,\sum_{l_r l'} \int d\,\vec{R}\,\sum_{\lambda_r \lambda'} (l_r\,\lambda_r\,l'\,\lambda' \mid l\,\lambda)\,\mathrm{Y}^{\lambda'*}_{l'}\,(\vec{R})$$

$$\times\,\mathcal{Y}^{\lambda_r *}_{l_r}\,(i\,\vec{p})\,\mathrm{F}_{l\,l_r\,l'}\,(\mathrm{R})\,\chi^{(-)*}_{\vec{k}_b}\,(\gamma\,\vec{R})\,\chi^{(+)}_{\vec{k}_a}\,(\vec{R}) \tag{6}$$

where $F_{l\,l_r\,l'}$ (R) is the form factor containing the nuclear structure information and $\mathcal{Y}^{\lambda_r}_{l_r}$ is the solid spherical harmonic of order l_r.

In this asymptotic approximation the form factor $F_{l\,l_r\,l'}$, takes a very simple form. The term in eq. (6) corresponding to $l_r = 0$ is then the well known Buttle and Goldfarb approximation. The other terms in eq. (6) contain the effects of recoil and their form factors are given in ref. [1].

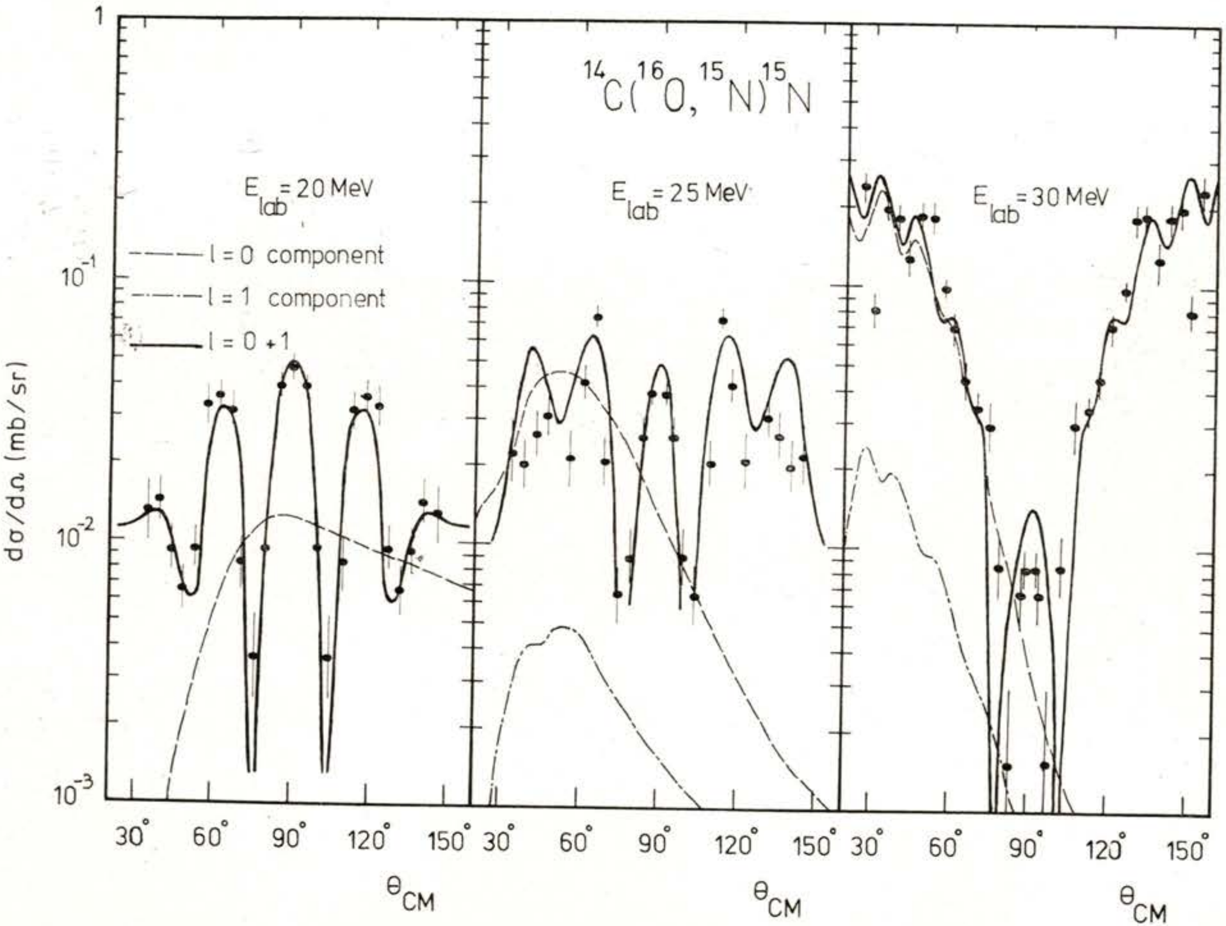

Fig. 1 — **DWBA** calculations for the $^{14}C(^{16}O,^{15}N)^{15}N$ ground state transition for three laboratory incident energies of 20, 25 and 30 **MeV** including recoil effects and showing the contributions from the allowed values of the total angular momentum transfer $l = 0$ and 1. The broken curves correspond to the direct transition amplitudes and the full curves result from antisymmetrization. The data are from ref. [7].

We have performed DWBA finite range calculations shown in Fig. 1 for the ^{14}C (^{16}O, ^{15}N) ^{15}N reaction at $E_{lab} = 20$, 25 and 30 MeV where the recoil effects were taken into account using the asymptotic approximation. The optical model potentials are from ref. [5, 6].

We find that the resulting spectroscopic factors are very close to those derived from a full finite range calculation using the DWBA code LOLA [7].

The approximation where the bound state wave functions are represented by the asymptotic Hankel functions is not essential for the application of the formalism which has been developed. It is possible to give a more realistic description of the bound states as a sum of Hankel functions [8] and still retain the simplicity of the above DWBA calculation.

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