AN EPITOME OF CONFIGURATIONAL DATA ON CONNECTED CLUSTERS

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ABSTRACT — New lattice data on configurational histograms are given for bond and site clusters grouped by fixed percolation perimeter, fixed energy perimeter and fixed cluster size. The latter are illustrated by several combinations of interest of cyclomatic number discriminations.

INTRODUCTION

It has long been recognized that configurational studies are a fundamental tool in the theory of critical phenomena. Recently, however, powerful techniques (like transfer-matrix renormalization and field theoretical methods [17]) have surged on to the statistics of lattice clusters in the percolation and animal problems (see e. g. ref [18]) and significant advances in the knowledge of the critical exponents for both problems have been brought close to a virtually «exact» solution. There is, however, still open a rich field of specializations (valence, cyclomatic number, specific connectivity requirements, restricted sets of clusters (animals) defined through topological constraints). Our aim in this paper is twofold: written in mid-81 it should concentrate on selected topics referring to the cluster topology which are likely to assume physical relevance in the future, and where series expansions and configurational studies will remain competitive. On the other hand, it should unify various treatments that have remained scattered in the literature without any systematic exploration (like bond or site content in percolation). We have divided the data in 5 broad groups: fixed energy groupings, fixed percolation perimeter groupings, fixed size percolation groupings, cyclomatic number distributions (in percolation) and fixed size energy groupings. Each one of them is preceded by a succint description of the graph theoretical procedures used in its derivation.

The notation to be consistently applied throughout the paper is:

s – denotes the number of cluster sites
b – denotes the number of cluster bonds
$c = b - s + 1$ denotes the cyclomatic number of a connected cluster
e – denotes the external bond (energy) perimeter
t – denotes the perimeter in the percolation sense

g_{se}; g_{sbt} – give the number of geometrically different cluster configurations with a given label *s,e* or *s,b,t*.

Note that the normalization of the various $g_{s\ldots}$ may occasionally vary for convenience. We have indicated in each case the factor relative to a normalization per lattice site.

A — *Fixed energy groupings*

Whenever the bond perimeter of connected site clusters is fixed, the resulting distributions according to the variable number of sites enclosed within a given configuration of boundary bonds can easily be translated topologically into a fixed perimeter — enclosed area problem by considering the dual lattice (Sykes et al [1], [2]). Consider figure 1 for the triangular — honeycomb system: in Fig. 1 A, the connected cluster of 8 sites and 11 bonds on the triangular lattice is the *dual* of the honeycomb configuration with 26 sides and area 8. Denoting the number of sites by *s*, the number of *(internal)* bonds by *b* and external bonds by *e*, the following linkage rule for site clusters (strongly embedded clusters)

$$e = zs - 2b \qquad \text{A. 1}$$

is valid on any lattice (coordination number z). For configurations of the type in Fig. 1A there are no sites enclosed within the configuration and not belonging to it but Fig. 1B indicates the possibility of such configurations: all clusters that can be derived from the fully compact cluster limited by the outermost boundary through the exclusion of any combination of hexagonal faces marked with (x) are still duals of connected clusters on the triangular lattice, but, unlike the case of Fig. 1A, their boundary is no longer singly-connected (it is no longer a simple polygon).

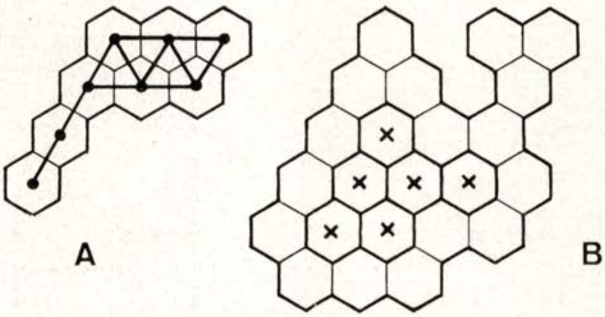

Fig 1

A — Site cluster on the triangular lattice and its dual on the honeycomb lattice. The triangular configuration is compact (no inner perimeter sites) and its dual is bounded by a simple polygon.

B — Another example of a honeycomb configuration. Exclusion of any face marked with (x) generates a connected dual from the larger simple polygon.

The situation recurs for the simple quadratic lattice, which is well known to be self-dual (Fig. 2). Fig. 2C is a compact configuration bounded by a simple polygon (it is, in fact, the isoperimetric solution for perimeter 18, Duarte and Marques [3] — and area 20). Once again, exclusion of any combination of square faces marked with (x) generates a connected area (alternative examples are drawn in 2A and 2B), which is still a dual of some site cluster on the same lattice. Fig. 2D shows, explicitly, a square site tree (17 sites) and its connected dual.

Now all perimeter distributions of site clusters contribute to the low temperature ferromagnetic polynomials for the Ising model [1]. It is, however, required, for their isolation, to separate the contribution of multicomponent graphs as described in [4]. To go one stage further and separate the simple polygons from the nonpolygonal connected duals, we note that on the honeycomb lattice (Fig. 1) it is impossible to have more than 3 elementary hexagonal faces meeting at a site and, therefore, the contribution from those configurations can be singled out from clusters discriminations on the triangular lattice taking into account the number of elementary triangular faces *f* (as well as *s* and *b*). Isolated inner boundary sites will then occur for all clusters where *f* does not account for the total cyclomatic number:

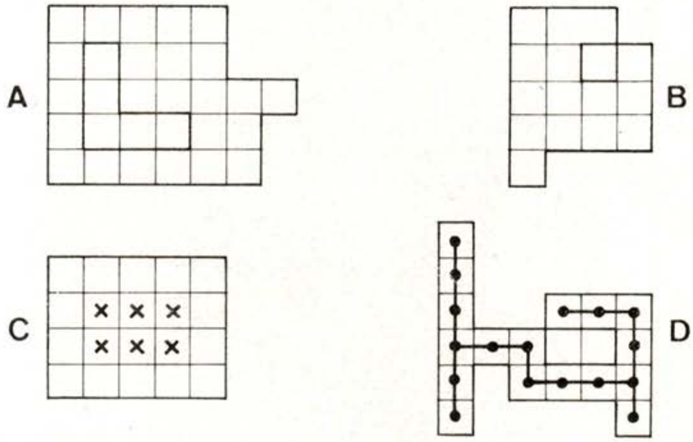

Fig. 2

A, B — Examples of connected duals on the square lattice.
C — Simple polygon on the square lattice. Exclusion of any face marked with (x) generates a connected dual from the larger configuration.
D — A square lattice tree and its dual.

$$b - s + 1 \neq f \qquad \text{A. 2}$$

and this inequality identifies the non-polygonal connected duals: all inner boundaries will be separate from the outermost boundary.

It is impossible to proceed in this way for the square lattice: a tree like in Fig. 2D does not verify A.2 and yet its dual is not a polygon. In general the problem of determining the connected duals up to a reasonable order is lessened by simple conversion of fixed *s* distributions (*b* groupings), like those below in section D. Earlier results for the square polygons can be found in Hiley and Sykes [5].

The additional data should sum to the known results for the total number of polygons (fixed perimeter), greatly extended in a recent paper by Enting [6] through the use of transfer matrix techniques.

We give new data for polygons on the honeycomb ($e \leq 42$) and square lattices ($e \leq 22$) as well as for the corresponding connected duals.

B — *Fixed percolation perimeter groupings*

When the perimeter is measured in the percolation sense, i.e. by the number of sites (bonds) *t* necessary for the isolation of a given cluster on a lattice, the perimeter groupings suffer considerable rearrangement of the various cluster contributions. The usual perimeter method can, of course, be used for obtaining these groupings — once again, they can be obtained through a straightforward conversion of the fixed size percolation distributions, although such information must be completed (for detailed descriptions see Sykes et al [4], Blease et al [7]).

In this paper we present results for these groupings on the square ($t \leq 16$) and honeycomb ($t \leq 13$) lattices (site problem) as well as for the Kagomé site problem ($t \leq 16$). As in section A, the problem is equivalent to the enumeration of the histograms g_{st} or g_{bt} at fixed *t*; g_{st} or g_{bt} gives the number of geometrically different cluster configurations per site (or per bond) with a given perimeter value *t* (here *t* refers to bond and site perimeter for bond and site percolation respectively).

In addition, we have used inequality A.2 (and further discrimination through *b*, *s*, *t* and *f*) to isolate all the non-polygonal connected duals of the triangular lattice according to their percolation weight. The resulting perimeter polynomials are given through order 21 (they should be compared with the complete set of perimeter groupings for the problem, given in Sykes et al [4] ($t \leq 22$)).

C — *Fixed size percolation groupings*

The g_{st} (fixed size *s*) are the best illustrated groupings in the literature. They have been listed for 2 , 3 and higher dimensions [8], [9], for both site and bond [10] problems on most usual lattices. The interested reader should refer to those papers for an outline of the method and detailed considerations on the applicability of the corresponding series expansions. We have added in this paper the groupings for the site problem on the 2 archimedean lattices of coordination number 5, (3,3,3,4,4) and (3,3,4,3,4) ($s \leq 12$). Both lattices (their Ising points are known exactly) provide good testing ground for the variation of the perimeter — to — size ratio and its connection with criticality (Duarte [11]). The well known sum rule to be verified by the g_{st} is

$$p = \sum_{s,t} s \; g_{st} \; p^s \; (1 - p)^t \qquad \text{C. 1}$$

D — *Cyclomatic number distributions in percolation*

A different type of configurational weighting which has been the object of much recent interest is the set of three – indexed discriminations of clusters by their site, bond content and perimeter (in the percolation sense). Through Euler's law these discriminations lead to the expansions of the average cyclomatic number $< c >$ (Cherry [12], Gaunt et al [13]).

$$< c > = < b > - < s > + < 1 > \qquad \text{D. 1}$$

and from expansion of the higher moments of the cluster size distribution of the type

$$< s^k > = \sum_{s,b,t} s^k \; g_{sbt} \; p^b \; (1 - p)^t \qquad \text{D. 2}$$

for bond percolation and

$$< b^k > = \sum_{s,b,t} b^k \, g_{sbt} \, p^s \, (1 - p)^t \qquad D.\ 3$$

for site percolation, new quantities of interest, paralleling the moments in the usual cluster size distribution are obtained. They are expected to belong to the same universality class as normal percolation and therefore constitute alternative ways of calculating the critical exponents for percolation. For $k = 2$, D.2 and D.3 lead to «susceptibility» series diverging near p_c with a critical exponent γ, like the mean cluster size

$$p \to p_c^- , < s^2 >_{bond} \sim | p_c - p |^{-\gamma} , < b^2 >_{site} \sim | p_c - p |^{-\gamma}, \qquad D.\ 4$$

This property has been occasionally used in the literature (Dunn et al [14], Agrawal et al [15]). A systematic study for 2 dimensional percolation (p_c lattices) is reported in [16].

We present results for the set of histograms $\Sigma_b \, b g_{sbt}$ for the triangular, square matching, Kagomé, honeycomb and archimedean (3,3,3,4,4) and (3,3,4,3,4) site problems and for $\Sigma_s \, s g_{sbt}$ for the square and honeycomb bond problems, as well as for $\Sigma_b \, b^2 \, g_{sbt}$ for the Kagomé site problem. We recall that for the first moment distributions the sum rules

$$\sum_{b,s,t} b \, g_{sbt} \, p^s \, (1 - p)^t = (z/2) \, p^2 \qquad D.\ 5$$

for site percolation and

$$\sum_{b,s,t} s \, g_{sbt} \, p^b \, (1 - p)^t = 1 - (1 - p)^z \qquad D.\ 6$$

for bond percolation, should be verified.

It also seems adequate to mention that the use of detailed valence discriminations constitute an alternative way of determining their cyclomatic number distributions. Since they represent an expansion from 3-indexed to z-indexed discriminations it is usually more cumbersome to take this line of procedure (it was however followed in refs [12], [13]). If the sites in a connected cluster are partitioned according to the number of site neighbours in the cluster (their valence) the following linkage rules are verified

$$\sum_{v} s_v = s \qquad \text{D. 7}$$

$$\sum_{v} v\, s_v = 2\, b \qquad \text{D. 8}$$

with s_v the number of sites with valence v and where the summations run from 1 to z. Equations D. 7 and D. 8 are unwieldy for bond percolation (valence considerations apply equally to bond and site percolation clusters), since b is usually fixed and any cyclomatic number mixing in a given g_{bt} must be disentangled from a combination of different s as well as from all compatible combinations of s_v's. It is generally more direct to start from bond percolation distributions and exploit the following properties of the yield factor generation (see Blease et al [7] for an exposition of the method):

a) For a given space-type strongly embeddable on the specific lattice under investigation, the number of bonds that can be transferred from the bond content to the bond perimeter is not greater than the cyclomatic number, if connectivity is to be preserved.

b) The number of transferable bonds is zero for strongly embedded trees.

c) The length of the bond tree percolation polynomial (with $b = s - 1$ bonds) is $c_{max} + 1$, where c_{max} is the maximum cyclomatic number of s-site clusters.

d) Strongly embedded clusters always maximise the bond perimeter for given values of b and c. Recalling that the linkage rule for strongly embedded clusters is $e = t = zs - 2\,b$, it can be seen that the difference in bond perimeter between clusters with sucessive cyclomatic numbers (same b) is z. These properties enable a separation of the cyclomatic number contributions in bond perimeter polynomials of not too high b (like those in Sykes et al [10]). In every case the sum rule D. 6 acts as a check on such graph theoretical manipulations and use of valence discriminations is thereby avoided.

E — *Fixed size energy groupings*

These enumerations are converse of those considered in section A. For site clusters, these groupings constitute (through eq. A. 1) a strict partition according to the cyclomatic number (or alternatively, according to the number of bonds *b* in the cluster). No data exist in the literature regarding this specific partition, which has emerged in recent times as a very relevant tool for the study of branched polymers in the dilute limit (mainly through the studies of Lubensky and coworkers [17], [18]). Clearly, in this partition the 3-indexed discriminations in D. 2 and D. 3, the g_{sbt}, are summed over the perimeter index, so that with the resulting g_{sb} (which are equivalent to the g_{se}) new moments of the «animal distribution» [17] can be defined and numerically investigated.

We present data on the three 2-dimensional regular lattices. The first noticeable difference with respect to the g_{ts} is that the corresponding histograms evolve very slowly in shape, so that it might be argued that for this specific partition the lattices are not very effectively sampled. Now, in each case, the maximum bond perimeter corresponds to the minimum number of bonds in the cluster, so that the last value in each of the g_{se} histograms just gives the total number of site trees on each lattice. As we have mentioned, in the previous section, this total number will also appear as the maximum perimeter configuration value in the bond percolation polynomial of bond size $b = s - 1$. Hence, the present data represent an extension over the data of ref [10].

Unfortunately the following term in g_{se} (corresponding to the total number of polygons, tadpoles and other configurations of cyclomatic number 1) does not grow sufficiently fast for a non--degenerate histogram to occur for loose-packed lattices. Consideration of the bond case only worsens the balance of the histogram: A. 1 is no longer valid, so that the g_{sb} with $s = b + 1$, gives the total number of bond trees and through the use of the yield factor generation all site clusters of $b + 1$ sites give non-zero contributions to $g_{b+1, b}$.

In order to avoid these problems one must concentrate on high coordination number (site) lattices where the strong embeddability «propagates» the distributions towards lower *e* values. But, better still, the careful exploitation of local linkage rules and consideration of both site and bond valence and the changes they undergo in bond-to-site transformations have enabled Duarte and Ruskin [19] to identify the valence structure on lattices which are covering lattices of bond problems. For the site trees ($g_{b+1,b}$) are always neighbour avoiding walks, belonging to a totally different universality class from branched trees and with a comparatively smaller growth parameter. The same happens for the terms of the form g_{bb} which originate from the corresponding bond polygons and bond trees with one single site of valence 3. Hence the g_{se} for the corresponding site covering problem shows a rapid evolution towards non-degenerate configurational histograms. We illustrate this point with the square covering g_{se} ($s \leq 13$).

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APPENDIX

FIXED ENERGY GROUPINGS

A — Honeycomb polygons

e = 6	g_{es} (× 1)	e = 20	
1	1	5	60
e = 10		6	42
2	3	7	30
e = 12		8	6
3	2	e = 22	
e = 14		5	99
3	9	6	129
4	3	7	105
e = 16		8	69
4	12	9	27
5	6	10	3
e = 18		e = 24	
4	29	6	280
5	21	7	276
6	14	8	246
7	1	9	160

10	86
11	24
12	2
e = 26	
6	348
7	726
8	720
9	609
10	432
11	249
12	117
13	27
14	3
e = 28	
7	1242
8	1710
9	1812
10	1458
11	1164
12	702
13	414
14	168
15	42
16	6
e = 30	
7	1260
8	3759
9	4611
10	4769
11	3870
12	3163
13	2126
14	1320
15	729
16	290
17	87
18	14
19	1
e = 32	
8	5436
9	9804
10	12186
11	12030
12	10476
13	8406
14	6336
15	4134
16	2622

17	1368
18	606
19	198
20	42
21	6
e = 34	
8	4644
9	18786
10	27627
11	33405
12	32061
13	29097
14	23553
15	18597
16	13128
17	8877
18	5412
19	2943
20	1401
21	507
22	147
23	27
24	3
e = 36	
9	23472
10	54148
11	76662
12	88378
13	86860
14	78978
15	67134
16	53826
17	40866
18	29076
19	19672
20	12006
21	6936
22	3424
23	1458
24	496
25	128
26	24
27	2
e = 38	
9	17382
10	90924
11	160131
12	218436

13	240579	27	13128
14	242613	28	6060
15	219816	29	2412
16	193602	30	798
17	158682	31	216
18	126099	32	42
19	93948	33	6
20	68019	e = 42	
21	45531	10	65822
22	29049	11	431448
23	17115	12	897289
24	9138	13	1369834
25	4338	14	1691994
26	1719	15	1865164
27	579	16	1873893
28	147	17	1778925
29	27	18	1601354
30	3	19	1397388
e = 40		20	1168533
10	100740	21	951897
11	287838	22	742157
12	464580	23	564297
13	604434	24	410122
14	661206	25	288397
15	669792	26	192099
16	619944	27	122932
17	553584	28	73674
18	469290	29	41040
19	384144	30	21083
20	300192	31	9632
21	226296	32	3918
22	163500	33	1341
23	111960	34	392
24	73266	35	87
25	44646	36	14
26	25626	37	1

A — *Square polygons*

e = 4, e = 6, e = 8, e = 10, e = 12, e = 14	(see Hiley, Sykes [5])	9	656
		10	482
		11	310
		12	151
		13	68
e = 16	g_{se} (× 1)	14	22
7	566	15	6
8	676	16	1

e = 18	
8	1868
9	2672
10	2992
11	2592
12	2086
13	1392
14	864
15	456
16	218
17	88
18	30
19	8
20	2
e = 20	
9	6237
10	10376
11	13160
12	12862
13	11717
14	9332
15	7032
16	4748
17	3010
18	1728
19	914
20	426
21	187
22	68
23	22
24	6
25	1
e = 22	
10	21050
11	39824
12	56162
13	61032
14	60864
15	54032
16	45936
17	35952
18	26858
19	18744
20	12456
21	7648
22	4472
23	2408
24	1208
25	560
26	238
27	88
28	30
29	8
30	2

A — *Honeycomb duals*

e = 6, e = 10, e = 12, e = 14, e = 16, e = 18, e = 20, e = 22 — Same as polygons

e = 24	g_{se} (× 1)
6	281
7	276
8	246
9	160
10	86
11	24
12	2
e = 26	
6	348
7	732
8	720
9	609
10	432
11	249
12	117
13	27
14	3
e = 28	
7	1248
8	1737
9	1818
10	1458
11	1164
12	702
13	414

14	168
15	42
16	6
e = 30	
7	1260
8	3795
9	4697
10	4817
11	3876
12	3163
13	2126
14	1320
15	729
16	290
17	87
18	14
19	2
e = 32	
8	5472
9	9990
10	12453
11	12264
12	10557
13	8418
14	6336
15	4134
16	2622
17	1368
18	606
19	198
20	42
21	6
e = 34	
8	4644
9	19014
10	28305
11	34263
12	32901
13	29601
14	23721
15	18627
16	13128
17	8877
18	5412
19	2943
20	1401
21	507
22	147
23	27
24	3
e = 36	
9	23662
10	54411
11	78990
12	91389
13	89680
14	81183
15	68294
16	54261
17	40950
18	29083
19	19672
20	12006
21	6936
22	3424
23	1458
24	496
25	128
26	24
27	2
e = 38	
9	17382
10	92205
11	165144
12	226125
13	250641
14	252177
15	227970
16	199110
17	161712
18	127287
19	94242
20	68067
21	45531
22	29049
23	17115
24	9138
25	4338
26	1719
27	579
28	147
29	27
30	3
e = 40	
10	101679
11	295356

12	482850
13	631218
14	694122
15	702816
16	648951
17	575856
18	484095
19	392556
20	303741
21	227472
22	163743
23	111990
24	73266
25	44646
26	25626
27	13128
28	6060
29	2412
30	798
31	216
32	42
33	6
e = 42	
10	65822
11	438264
12	929414
13	1431751
14	1782600
15	1971774
16	1985091
17	1879842
18	1685370
19	1458954
20	1210437
21	975901
22	753821
23	568761
24	411402
25	288661
26	192123
27	122932
28	73674
29	41040
30	21083
31	9632
32	3918
33	1341
34	392
35	87
36	14
37	1

A — *Square duals*

e = 4	g_{se} ($\times 1$)
1	1
e = 6	
2	2
e = 8	
3	6
4	1
e = 10	
4	18
5	8
6	2
e = 12	
5	55
6	40
7	22
8	6
9	1
e = 14	
6	174
7	168
8	134
9	72
10	30
11	8
12	2
e = 16	
7	570
8	677
9	656
10	482
11	310
12	151
13	68
14	22
15	6
16	1
e = 18	
8	1908
9	2708
10	3008

11	2596
12	2086
13	1392
14	864
15	456
16	218
17	88
18	30
19	8
20	2
e = 20	
9	6473
10	10724
11	13456
12	13034
13	11789
14	9354
15	7036
16	4748
17	3010
18	1728
19	914
20	426
21	187
22	68
23	22
24	6
25	1
e = 22	
10	22202
11	42012
12	58742
13	63256
14	62396
15	54908
16	46352
17	36112
18	26906
19	18756
20	12456
21	7468
22	4472
23	2408
24	1208
25	560
26	238
27	88
28	30
29	8
30	2

PERCOLATION PERIMETER GROUPINGS

B — *Square lattice*

t = 4	g_{st} (× 1)
1	1
t = 6	
2	2
t = 7	
3	4
t = 8	
3	2
4	9
5	1
t = 9	
4	8
5	20
6	4
t = 10	
4	2
5	28
6	54
7	22
8	4
t = 11	
5	12
6	80
7	136
8	80
9	28
10	4
t = 12	
5	2
6	60
7	252
8	388
9	291
10	154

11	52
12	9
13	1
t = 13	
6	16
7	228
8	777
9	1152
10	986
11	644
12	325
13	112
14	28
15	4
t = 14	
6	2
7	100
8	818
9	2444
10	3676
11	3530
12	2644
13	1660
14	828
15	332
16	106
17	22
18	4
t = 15	
7	20
8	480
9	2804
10	7612
11	11772
12	12502
13	10480
14	7508
15	4608
16	2406
17	1104
18	396
19	124
20	28
21	4
t = 16	
7	2
8	152
9	2089
10	9750
11	24472
12	38694
13	44574
14	41408
15	33046
16	23311
17	14385
18	8146
19	3982
20	1730
21	651
22	206
23	52
24	9
25	1

B — *Honeycomb lattice*

t = 3	g_{st} (× 1)
1	1
t = 4	
2	1.5
t = 5	
3	3
t = 6	
4	7
5	3
6	0.5
t = 7	
5	15
6	15
7	3
t = 8	
6	31.5
7	60
8	37.5
9	12
10	1.5

t = 9	
7	62
8	177
9	190
10	111
11	39
12	9
13	1
t = 10	
8	123
9	471
10	744
11	705
12	449.5
13	207
14	69
15	15
16	1.5
t = 11	
9	246
10	1167
11	2361
12	3006
13	2721
14	1902
15	1083
16	492
17	162
18	33
19	3
t = 12	
10	503
11	2874
12	7078
13	11181
14	12937.5
15	11758
16	8895
17	5796
18	3258
19	1522
20	565.5
21	164
22	37
23	6
24	0.5
t = 13	
11	1029
12	6927
13	20160
14	37635
15	52311
16	57960
17	53949
18	43728
19	31536
20	20355
21	11689
22	5889
23	2541
24	894
25	234
26	39
27	3

B — *Kagomé lattice*

t = 4	g_{st} (× 1)
1	1
t = 5	
2	2
t = 6	
3	14/3
6	1/3
t = 7	
4	12
5	2
7	2
t = 8	
5	31
6	12
8	9
11	1
t = 9	
6	81 1/3
7	54
9	36 2/3
10	11

12	8
15	2/3
t = 10	
7	216
8	220
9	35
10	133
11	88
13	45
14	10
16	9
19	1
t = 11	
8	576
9	798
10	280
11	454
12	496
13	58
14	212
15	138
17	66
18	19
20	14
23	2
t = 12	
9	1550 1/3
10	2724
11	1576
12	1620
13	2344
14	804
15	877
16	1020
17	153
18	371
19	270
21	118 2/3
22	46
24	29
27	4 2/3
30	1/3
t = 13	
10	4210
11	8940
12	7432
13	6536
14	9726
15	5952
16	3974
17	5770
18	2190
19	1780
20	2318
21	468
22	748
23	674
25	258
26	126
28	70
29	10
31	14
34	2
t = 14	
11	11522
12	28524
13	30774
14	28078
15	38372
16	33712
17	21063
18	27850
19	18925
20	10303
21	14744
22	6903
23	3946
24	6058
25	1542
26	1729
27	1904
28	139
29	639
30	430
32	195
33	68
35	49
38	9
41	1
t = 15	
12	317770 2/3
13	89636
14	117736
15	120652 2/3
16	151052
17	162910

18	119496
19	128790
20	120932
21	69820 ⅔
22	78394
23	62468
24	29484 ⅔
25	40976
26	23430
27	10837 ⅓
28	17521
29	6096
30	4432
31	6050
32	1082
33	1763 ⅓
34	1689
36	612
37	342
39	179 ⅓
40	41
42	42 ⅔
45	8
48	2/3
t = 16	
13	88129
14	279000
15	427488
16	500865
17	608253
18	716926
19	637131
20	608309
21	645568
22	472456
23	418304
24	425165
25	247522
26	237541
27	216930
28	99805
29	122492
30	86467
31	36226
32	55156
33	27197
34	12362
35	21294
36	681
37	5339
38	6868
39	800
40	2052
41	1778
43	693
44	360
46	203
47	32
49	49
52	9
55	1

B — *Triangular lattice (without holes)*

t = 6	g_{st} (× 1)
1	1
t = 8	
2	3
t = 9	
3	2
t = 10	
3	9
4	3
t = 11	
4	12
5	6
t = 12	
4	29
5	21
6	14
7	1
t = 13	
5	66
6	42
7	30
8	6
t = 14	
5	93
6	153

7	105
8	69
9	27
10	3
t = 15	
6	298
7	360
8	264
9	160
10	86
11	24
12	2
t = 16	
6	306
7	840
8	918
9	717
10	444
11	249
12	117
13	27
14	3
t = 17	
7	1290
8	2316
9	2382
10	1884
11	1308
12	720
13	414
14	168
15	42
16	6
t = 18	
7	1014
8	4299
9	6486
10	6641
11	5160
12	3913
13	2354
14	1356
15	729
16	290
17	87
18	14
19	1
t = 19	
8	5310
9	13488
10	18006
11	18732
12	14892
13	11310
14	7764
15	4614
16	2712
17	1374
18	606
19	198
20	42
21	6
t = 20	
8	3408
9	20469
10	41673
11	51822
12	52992
13	45129
14	33981
15	24900
16	16188
17	10023
18	5676
19	2879
20	1401
21	507
22	147
23	27
24	3
t = 21	
9	21372
10	71644
11	125448
12	153614
13	152658
14	136014
15	106416
16	79446
17	55440
18	36576
19	22708
20	12912
21	7116

22	3442	25	128
23	1458	26	24
24	496	27	2

SIZE PERCOLATION GROUPINGS

C — *Archimedean (3, 3, 4, 3, 4) (site problem)*

s = 4	g_{st} (× 4)	14	2926
5	4	15	6680
s = 2		16	8164
6	2	17	4632
7	8	18	848
s = 3		19	32
8	20	s = 9	
9	12	12	104
s = 4		13	612
8	2	14	2960
9	28	15	8780
10	66	16	20116
11	16	17	29908
s = 5		18	25312
9	8	19	9888
10	48	20	1266
11	180	21	36
12	156	s = 10	
13	20	12	24
s = 6		13	372
10	28	14	1998
11	108	15	9156
12	432	16	27284
13	676	17	62016
14	304	18	103726
15	24	19	110440
s = 7		20	68554
10	4	21	19204
11	72	22	1836
12	316	23	40
13	1092	s = 11	
14	2180	12	8
15	1928	13	128
16	528	14	1308
17	28	15	7104
s = 8		16	28436
11	20	17	86612
12	204	18	198268
13	988	19	350032

20	437312
21	356728
22	166292
23	34668
24	2556
25	44
s = 12	
12	2
13	40
14	660
15	4816
16	24572
17	91972
18	279622
19	645560
20	1183662
21	1639860
22	1606806
23	1032628
24	368306
25	59220
26	3452
27	48

C — *Archimedean (3, 3, 3, 4, 4) (site problem)*

s = 1	g_{st} (× 2)
5	2
s = 2	
6	2
7	2
8	1
s = 3	
7	2
8	4
9	10
s = 4	
8	2
9	10
10	33
11	10
12	2
s = 5	
9	2
10	34
11	72
12	68
13	36
s = 6	
10	7
11	90
12	172
13	254
14	254
15	36
16	4
s = 7	
11	34
12	204
13	504
14	832
15	1084
16	464
17	110
s = 8	
11	4
12	120
13	504
14	1523
15	2750
16	3791
17	2906
18	1294
19	118
20	8
s = 9	
12	36
13	346
14	1512
15	4496
16	9004
17	13046
18	13130
19	8992
20	2300
21	314
s = 10	
12	5
13	158
14	1080
15	4758

16	13604	23	55766
17	28980	24	9696
18	45462	25	854
19	52606	s = 12	
20	45831	13	4
21	21304	14	247
22	5518	15	2310
23	358	16	12693
24	16	17	48018
s = 11		18	137623
13	42	19	306124
14	608	20	541719
15	3670	21	755720
16	14956	22	828850
17	42802	23	676978
18	93434	24	392305
19	157478	25	123404
20	202272	26	21028
21	200886	27	1024
22	133464	28	32

CYCLOMATIC NUMBER DISTRIBUTIONS IN PERCOLATION

D — *Triangular lattice*

s = 2	$\Sigma_b\ bg_{sbt}$ (× 1)	s = 7	
8	3	12	12
s = 3		13	330
9	6	14	1098
10	18	15	3198
s = 4		16	6504
10	15	17	8802
11	48	18	6084
12	87	s = 8	
s = 5		13	84
11	42	14	897
12	126	15	3420
13	324	16	10230
14	372	17	22494
s = 6		18	37251
12	126	19	41430
13	342	20	23856
14	1047	s = 9	
15	1746	14	432
16	1530	15	2478
		16	10962

17	32550
18	77244
19	142686
20	196716
21	187836
22	92496
s = 10	
14	57
15	1548
16	8256
17	33522
18	107844
19	258930
20	528585
21	822762
22	991227
23	826092
24	356391
s = 11	
15	504
16	5088
17	28686
18	106518
19	354294
20	882774
21	1886160
22	3269094
23	4492410
24	4797486
25	3559656
26	1366230
s = 12	
15	48
16	2691
17	17442
18	97686
19	358230
20	1154871
21	3044358
22	6663264
23	12435252
24	19008123
25	23432184
26	22557609
27	15089856
28	5218950
s = 13	
16	702
17	10626
18	67356
19	322320
20	1241400
21	3838218
22	10430136
23	23661498
24	46122252
25	76516392
26	105213888
27	118095138
28	103613454
29	63161082
30	19879872
s = 14	
16	87
17	4704
18	40776
19	254682
20	1112547
21	4240902
22	13165455
23	35992752
24	83918157
25	169687884
26	297846975
27	446213892
28	560840964
29	578723592
30	467147454
31	261616854
32	75562266

D — *Square matching site problem*

	s = 2	$\Sigma_b\, bg_{sbt}$ ($\times$ 1)
10		2
12		2
	s = 3	
12		16
14		16
15		8
16		4
	s = 4	
12		6
14		78
15		32
16		96
17		64
18		72
19		24
20		6
	s = 5	
14		64
15		24
16		332
17		336
18		568
19		452
20		612
21		336
22		208
23		48
24		8
	s = 6	
14		22
16		396
17		476
18		1422
19		2164
20		3682
21		3064
22		4178
23		3264
24		2338
25		1180
26		460
27		80
28		10

	s = 7	
16		292
17		224
18		1960
19		4120
20		7968
21		12092
22		21732
23		22660
24		26520
25		24704
26		21956
27		13352
28		7820
29		3096
30		840
31		120
32		12
	s = 8	
16		100
17		72
18		1986
19		3872
20		10910
21		24892
22		51134
23		73640
24		119744
25		153072
26		177006
27		172104
28		168700
29		127632
30		86776
31		46364
32		21414
33		6580
34		1400
35		168
36		14
	s = 9	
16		20
18		1360
19		2528
20		11440

21	32760
22	75996
23	146044
24	304820
25	473628
26	700344
27	942580
28	1158604
29	1203972
30	1200980
31	1031344
32	810160
33	519744
34	294132
35	135616
36	49920
37	12224
38	2176
39	224
40	16
s = 10	
18	670
19	968
20	10406
21	31096
22	82660
23	219324
24	510996
25	948612
26	1795748
27	2903344
28	4335128
29	5806280
30	7281978
31	8178288
32	8407562
33	7738904
34	6627886
35	4895912
36	3241420
37	1800604
38	879220
39	341348
40	101672
41	21060
42	3204
43	288
44	18
s = 11	
18	208
19	372
20	7300
21	21692
22	81332
23	247844
24	646804
25	1492640
26	3236880
27	6180136
28	11020012
29	17637232
30	26672100
31	36246008
32	45860536
33	53549768
34	57479148
35	56049244
36	50716588
37	41178380
38	30463740
39	19695760
40	11292320
41	5528144
42	2318500
43	756984
44	190168
45	33960
46	4520
47	360
48	20

D — *Kagomé lattice site problem*

	$\Sigma_b \; bg_{stb}$ ($\times$ 3)
s = 2	
5	6
s = 3	
6	30
s = 4	
7	120
s = 5	
7	24
8	426
s = 6	
6	6
8	204
9	1416
s = 7	
7	48
9	1140
10	4548
s = 8	
8	258
10	5496
11	14220
s = 9	
9	1206
10	936
11	23052
12	43860
s = 10	
9	378
10	4932
11	8880
12	88956
13	134250
s = 11	
8	42
10	3504
11	18636
12	56616
13	325044
14	408894
s = 12	
9	384
11	21972
12	71826
13	296724
14	1141584
15	1241916
s = 13	
10	2376
11	2682
12	113640
13	306876
14	1349736
15	3912288
16	3762834
s = 14	
10	546
11	12126
12	40842
13	511212
14	1397292
15	5612268
16	13185468
17	11375370
s = 15	
9	42
11	8190
12	53886
13	329904
14	2159004
15	6383382
16	21974364
17	43892388
18	34321086
s = 16	
10	612
12	65484
13	255342
14	2012856
15	9002940
16	28199694
17	82526616
18	144709680
19	103371816
s = 17	
11	4824
12	10158

13	396048	17	118643394
14	1397940	18	301417152
15	10394394	19	473127060
16	38126082	20	310901274

D — *Honeycomb lattice site problem*

s = 2	$\Sigma_b\ bg_{stb}$ (× 2)
4	3
s = 3	
5	12
s = 4	
6	42
s = 5	
6	24
7	120
s = 6	
6	6
7	150
8	315
s = 7	
7	42
8	720
9	744
s = 8	
8	552
9	2478
10	1722
s = 9	
8	216
9	3126
10	7536
11	3936
s = 10	
8	33
9	2166
10	13623
11	21006
12	9054
s = 11	
9	882
10	14862
11	47766
12	57480
13	20580

s = 12	
9	234
10	10797
11	68670
12	156972
13	152394
14	46530
s = 13	
9	30
10	5616
11	69870
12	275772
13	486714
14	395352
15	104496
s = 14	
10	2097
11	54420
12	354312
13	998700
14	1457295
15	1001676
16	234312
s = 15	
10	510
11	34248
12	355644
13	1524900
14	3428742
15	4197072
16	2500512
17	524244
s = 16	
10	57
11	17130
12	295326
13	1849878
14	6028185
15	11161506

16	11757504
17	6162480
18	1171950
s = 17	
11	6198
12	209940
13	1876770
14	8488092
15	22152534
16	34968960
17	32137488
18	15050304
19	2616288
s = 18	
11	1386
12	128046
13	1650396
14	10041624
15	35364792
16	77356395
17	105800778
18	86255844
19	36453678
20	5835012

s = 19	
11	138
12	64644
13	1285596
14	10333392
15	47614188
16	137698308
17	258202302
18	311675004
19	227833902
20	87704424
21	12997152
s = 20	
12	25860
13	892020
14	9492018
15	55892538
16	206922024
17	506625396
18	832200162
19	896996238
20	593959458
21	209710524
22	28922142

D — *Archimedean lattice (3, 3, 4, 3, 4) site problem*

s = 2	$\Sigma_b \, bg_{stb}$ (× 4)
6	2
7	8
s = 3	
8	44
9	24
s = 4	
8	8
9	104
10	210
11	48
s = 5	
9	48
10	248
11	832
12	644
13	80
s = 6	
10	208
11	724

12	2652
13	3680
14	1560
15	120
s = 7	
10	36
11	656
12	2644
13	8220
14	15100
15	12272
16	3240
17	168
s = 8	
11	224
12	2190
13	9696
14	26442
15	55872
16	63322

17	33960	14	21148
18	6056	15	107820
19	224	16	407184
s = 9		17	1168464
12	1344	18	2518668
13	7436	19	4211012
14	33664	20	5012660
15	92464	21	3902412
16	196812	22	1753036
17	275136	23	357472
18	218848	24	25948
19	82256	25	440
20	10396	s = 12	
21	288	12	38
s = 10		13	768
12	356	14	12156
13	5408	15	84764
14	27408	16	409734
15	117676	17	1453032
16	327452	18	4178896
17	697796	19	9142376
18	1100140	20	15939366
19	1110356	21	21095484
20	656814	22	19821760
21	178864	23	12241136
22	16796	24	4242200
23	360	25	669848
s = 11		26	38508
12	136	27	528
13	2140		

D — *Archimedean lattice (3, 3, 3, 4, 4) site problem*

s = 2	$\Sigma_b\ bg_{stb}$ (× 2)	s = 5	
6	2	9	14
7	2	10	184
8	1	11	318
s = 3		12	280
7	6	13	144
8	8	s = 6	
9	20	10	56
s = 4		11	626
8	10	12	1028
9	38	13	1378
10	102	14	1292
11	30	15	180
12	6	16	20

s = 7	
11	322
12	1724
13	3826
14	5708
15	6828
16	2832
17	660
s = 8	
11	48
12	1322
13	5008
14	13914
15	22892
16	29028
17	21208
18	9158
19	826
20	56
s = 9	
12	488
13	4316
14	17404
15	47576
16	88356
17	118700
18	112864
19	74098
20	18600
21	2512
s = 10	
12	81
13	2390
14	15100
15	61894
16	163962
17	326878
18	480335
19	524380
20	435309
21	196808
22	50046
23	3222
24	144
s = 11	
13	754
14	10084
15	56894
16	216292
17	579562
18	1189958
19	1893344
20	2301600
21	2177084
22	1396480
23	569130
24	97684
25	8540
s = 12	
13	84
14	4770
15	41758
16	215426
17	764342
18	2066141
19	4346780
20	7293506
21	9678184
22	10131540
23	7961512
24	4476120
25	1382338
26	232628
27	11264
28	352

D — *Square bond problem*

s = 1	$\Sigma_s \, sg_{stb}$ (× 2)
6	4
s = 2	
8	18
s = 3	
9	16
10	72
s = 4	
8	4
11	160
12	275

s = 5	
10	40
12	180
13	960
14	1044
s = 6	
11	84
12	240
14	2324
15	4704
16	3990
s = 7	
10	12
13	1092
14	1176
15	2688
16	16240
17	21696
18	15264
s = 8	
12	154
14	1824
15	7664
16	7144
17	36576
18	89748
19	97920
20	58257
s = 9	
13	480
14	1072
15	1476
16	24984
17	42516
18	84992
19	273920
20	460040
21	432290
22	222020
s = 10	
12	48
14	558
15	6552
16	5904
17	53080
18	188160
19	281096
20	809612
21	1636008
22	2281884
23	1867624
24	845746

D — *Honeycomb bond problem*

s = 1	$\Sigma_s\ sg_{stb}$ (× 1)
4	6
s = 2	
5	18
s = 3	
6	56
s = 4	
7	180
s = 5	
7	36
8	558
s = 6	
6	6
8	252
9	1708
s = 7	
7	42
9	1296
10	5184
s = 8	
8	216
10	5940
11	15552
s = 9	
9	990
10	1050
11	23940
12	46510
s = 10	
9	330
10	3990
11	9240
12	89892
13	138930
s = 11	
8	30

10	2904
11	14982
12	56736
13	321840
14	414792
$s = 12$	
9	264
11	17856
12	58712
13	289848
14	1112436
15	1239056
$s = 13$	
10	1620
11	2262
12	91416
13	260268
14	1292508
15	3764712
16	3701418
$s = 14$	
10	390
11	8268
12	33768
13	408492
14	1218804
15	5298120
16	12555000
17	11054610
$s = 15$	
9	26
11	5796
12	36834
13	267840
14	1730016
15	5652390
16	20519424
17	41415840
18	33004544
$s = 16$	
10	378
12	45900
13	180702
14	1618176
15	7296948
16	25115160
17	76400448
18	135462936
19	98498952
$s = 17$	
11	2970
12	7344
13	276960
14	1037808
15	8308410
16	31418934
17	105743982
18	277089606
19	439782480
20	293866272

D — *Kagomé lattice site problem*

$s = 2$	$\Sigma_b \, b^2 g_{sbt}$ $(\times 3)$
5	6
$s = 3$	
6	66
$s = 4$	
7	408
$s = 5$	
7	96
8	1980
$s = 6$	
6	36
8	1164
9	8316
$s = 7$	
7	384
9	8088
10	32232
$s = 8$	
8	2472
10	46104
11	117972
$s = 9$	
9	13266
10	8406
11	223344
12	416466

s = 10	
9	4344
10	61158
11	94416
12	973692
13	1435470
s = 11	
8	588
10	46632
11	255828
12	681552
13	3958824
14	4861974
s = 12	
9	6144
11	325416
12	1066932
13	3967332
14	15296832
15	16256316
s = 13	
10	41856
11	41490
12	1841832
13	4835928
14	19813620
15	57146988
16	53773968
s = 14	
10	9954
11	231498
12	693798
13	8980896
14	23326068
15	89507916
16	208467996
17	176236950
s = 15	
9	882
11	162306
12	1105230
13	6111624
14	40617600
15	113174922
16	377802300
17	746740380
18	572993958
s = 16	
10	13878
12	1403460
13	5488974
14	40162224
15	179523264
16	531568206
17	1520199504
18	2636009496
19	1850067696
s = 17	
11	117576
12	225234
13	9075648
14	31074372
15	221592582
16	799876968
17	2375467950
18	5918963946
19	9187911360
20	5937914886

FIXED SIZE ENERGY GROUPINGS

E — *Honeycomb lattice*

s = 1	g_{se} (× 2)
3	2
s = 2	
4	3
s = 3	
5	6
s = 4	
6	14
s = 5	
7	36
s = 6	
6	1
8	93
s = 7	
7	6
9	244

s = 8	
8	27
10	648
s = 9	
9	110
11	1728
s = 10	
8	3
10	399
12	4651
s = 11	
9	24
11	1362
13	12630
s = 12	
10	135
12	4468
14	34566
s = 13	
9	2
11	636
13	14244
15	95312
s = 14	
10	27
12	2631
14	44706
16	264387
s = 15	
11	198
13	10050
15	138938
17	736974
s = 16	
10	3
12	1113
14	36405
16	428955
18	2062784
s = 17	
11	42
13	5340
15	127170
17	1318254
19	5794056
s = 18	
12	356
14	22941
16	433941
18	4035356
20	16325904
s = 19	
11	6
13	2244
15	91622
17	1456674
19	12310578
21	46133390
s = 20	
12	87
14	11838
16	348120
18	4830054
20	37445847
22	130703016

E — *Square lattice*

s = 1	g_{se} (× 1)
4	1
s = 2	
6	2
s = 3	
8	6
s = 4	
8	1
10	18
s = 5	
10	8
12	55
s = 6	
10	2
12	40
14	174
s = 7	
12	22
14	168
16	570
s = 8	
12	6

14	134
16	677
18	1908
s = 9	
12	1
14	72
16	656
18	2708
20	6473
s = 10	
14	30
16	482
18	3008
20	10724
22	22202
s = 11	
14	8
16	310
18	2596
20	13456
22	42012
24	76886
s = 12	
14	2
16	151
18	2086
20	13034
22	58742
24	163494
26	268352
s = 13	
16	68
18	1392
20	11789
22	63256
24	250986
26	633748
28	942651
s = 14	
16	22
18	864
20	9354
22	62396
24	297262
26	1056608
28	2448760
30	3329608
s = 15	
16	6
18	456
20	7036
22	54908
24	317722
26	1359512
28	4401192
30	9436252
32	11817582
s = 16	
16	1
18	218
20	4748
22	46352
24	303068
26	1563218
28	6095764
30	18173796
32	36285432
34	42120340
s = 17	
18	88
20	3010
22	36112
24	276464
26	1603984
28	7477928
30	26922156
32	74496544
34	139297108
36	150682450

E — *Triangular lattice*

s = 1	g_{se} (× 1)
6	1
s = 2	
10	3
s = 3	
12	2
14	9

s = 4	
14	3
16	6
18	29
s = 5	
16	6
18	21
20	60
22	99
s = 6	
18	14
20	42
22	129
24	281
26	348
s = 7	
18	1
20	30
22	105
24	276
26	732
28	1248
30	1260
s = 8	
20	6
22	69
24	246
26	720
28	1737
30	3795
32	5472
34	4644
s = 9	
22	27
24	160
26	609
28	1818
30	4697
32	9990
34	19014
36	23662
38	17382
s = 10	
22	3
24	86
26	432
28	1458
30	4817
32	12453
34	28305
36	55411
38	92205
40	101679
42	65822
s = 11	
24	24
26	249
28	1164
30	3876
32	12264
34	34263
36	78990
38	165144
40	295356
42	438264
44	434784
46	251655
s = 12	
24	2
26	117
28	702
30	3163
32	10557
34	32901
36	91389
38	226125
40	482850
42	929414
44	1531383
46	2050899
48	1852892
50	969819
s = 13	
26	27
28	414
30	2126
32	8418
34	29601
36	89680
38	250641
40	631218
42	1431751
44	2845248
46	5093199

48	7761168
50	9484524
52	7876554
54	3762517
s = 14	
26	3
28	168
30	1320
32	6336
34	23721
36	81183
38	252177
40	694122
42	1782600
44	4157097
46	8736174
48	16309377
50	27275403
52	38620725
54	43453965
56	33417534
58	14680890

E — *Square covering site problem*

s = 1	g_{se} (× 2)
6	2
s = 2	
10	6
s = 3	
12	4
14	18
s = 4	
12	1
16	37
18	50
s = 5	
16	12
18	26
20	192
22	142
s = 6	
18	16
20	102
22	246
24	874
26	390
s = 7	
18	2
20	24
22	226
24	640
26	1826
28	3508
30	1086
s = 8	
20	6
22	82
24	344
26	1924
28	4035
30	10858
32	13259
34	2958
s = 9	
22	26
24	168
26	1076
28	3336
30	13512
32	25240
34	56634
36	47320
38	8134
s = 10	
22	4
24	58
26	580
28	2266
30	9360
32	29444
34	83758
36	152964
38	266710
40	163340
42	22050
s = 11	
24	16
26	236
28	1372
30	6260

32	23720
34	74098
36	222372
38	493862
40	869372
42	1169136
44	545580
46	60146
s = 12	
24	1
26	76
28	743
30	3704
32	17174
34	58860
36	208354
38	553224
40	1507761
42	2822608
44	4625299
46	4832436
48	1784168
50	162466
s = 13	
26	12
28	316
30	2122
32	11448
34	42428
36	170104
38	520236
40	1612728
42	3966084
44	9399652
46	15765404
48	23066864
50	19101104
52	5711504
54	440750

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