D-STATE AND NUCLEAR STRUCTURE EFFECTS IN (d , α) REACTIONS

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(Received 14 December 1983)

ABSTRACT —A general discussion is given of the effects of the α-particle D-state in (d , α) and (α , d) reactions. The dependence of the cross section and of the tensor analysing powers T_{2q} on the asymptotic D- to S-state ratio ρ in the α particle and on the spectroscopic amplitudes of two-nucleon cluster transfer is discussed using a plane wave peripheral model. It is shown that the T_{2q} in (d , α) reactions contain specific information on the α-particle D-state and also on the coherence properties of the two-nucleon states populated.

1 — INTRODUCTION

It is well known that the polarization observables of transfer reactions can be used to investigate the internal structure of composite particles. This property has been extensively applied to study the two and three body bound systems via the (d, p) [1, 2], (d , t) and (d , ^{3}He) [3, 4] reactions. Recently it was suggested by Santos et al. [5] that the tensor analysing powers of ($\vec{d}$, α) reactions display the effect of a relative D-state motion of two deuteron clusters in the α particle. This low energy (d , α) data is primarily sensitive to the parameter D_2 [1-6] which is closely related to the asymptotic D to S-state ratio ρ.

The calculations of ref. [5] used a very simplified reaction model based in the plane wave approximation and did not take into account the effect of L mixing in the transition amplitude to unnatural parity states. More recently full finite range DWBA calculations [7] have shown that the tensor analysing powers of $(\vec{d}, \alpha)$ reactions are specially sensitive to the L mixing in unnatural parity transitions. This effect can be used to study the coherence properties of the states populated and to determine the spectroscopic amplitudes corresponding to each L value. Furthermore it was realized [7, 8] that the interference between L mixing and D-state effects in the presently available $(\vec{d}, \alpha)$ tensor analysing power data makes it difficult to extract D_2 from the data.

The cross section of (α, d) and (d, α) reactions is also sensitive to the α-particle D-state. Nagarajan and Satchler [9] have shown that the D-state effects have a J-dependence which is qualitatively in agreement with the J-dependence observed in the cross section of ^{208}Pb (α, d) reactions [10]. This was previously interpreted as resulting from multistep processes [10]. To compare these two types of J-dependence we need a more complete understanding of the D-state effects in (d, α) reactions and in particular a realistic estimate of D_2.

Here we develop the DWBA theory of (α, d) and (d, α) reactions including both the S and D-state components of the α-particle. In section 2 the decomposition of the transition amplitude for two nucleon transfer reactions is performed. These results are then applied to the particular case of (α, d) and (d, α) reactions in section 3. In section 4 using a perturbative approach to generate the D-state component of the α-particle we calculate D_2 using gaussian wave functions and realistic tensor interactions. Finally in section 5 the special sensitivity of the tensor analysing powers to the L mixing and D-state effects is studied using a peripheral model for the transfer.

2 — TWO NUCLEON TRANSITION AMPLITUDE

We consider a transfer reaction A (a , b) B where $a = b + x$ and x is the transferred cluster. The transition amplitude for the reaction, scattering from momentum $\mathbf{k}_a$ to momentum $\mathbf{k}_b$ is

$$T = < B\, J_B\, M_B\, ,\, b\, s_b\, \sigma_b\, ;\, \mathbf{k}_b \,|\, T \,|\, A\, J_A\, M_A\, ,\, a\, s_a\, \sigma_a\, ;\, \mathbf{k}_a > \tag{1}$$

where J_A, s_a, J_B, s_b are the spins of A, a, B, b. Performing an expansion into terms with definite angular momentum transfer [11] we can write

$$T = \sum_{sJl} (J_A M_A J M_J | J_B M_B)\ (l \lambda s \sigma | J M_J)$$
$$(-1)^{s_b - \sigma_b} (s_a \sigma_a s_b - \sigma_b | s\ \sigma)\ B^{l\lambda}_{sJ} \tag{2}$$

where $(J_A M_A J M_J | J_B M_B)$ is the usual Clebsch-Gordan coefficient [12].

The amplitudes $B^{l\lambda}_{sJ}$ contain the reaction dynamics and transform under rotations like the conjugate of the spherical harmonic Y^{λ}_{l}. It is important to notice that the expansion (2) in the angular momentum transfer representation is model independent since it is based only on the transformation properties under rotations of states with definite angular momentum. Therefore it does not assume any approximations regarding, for instance, spin dependent forces in the entrance and exit channels, the internal structure of the nuclei involved in the reaction and the one-step or sequential transfer nature of the reaction mechanism.

We shall now particularize eq. (2) to two-nucleon transfer. In this case $a = b + 2$ and $B = A + 2$. To proceed with the analysis of the transition amplitude we consider a double-parentage decomposition of the state $J_B\ M_B$ [13]

$$|B\ J_B\ M_B> = \sum_{\eta A' J M_J} \mathcal{S}_J(\eta)\, |\eta\, J M_J>\, |A' J_{A'} M_{A'}>$$
$$(J_{A'} M_{A'} J M_J | J_B M_B) \tag{3}$$

where $\mathcal{S}_J(\eta)$ is the spectroscopic amplitude for the η, J configuration of the two nucleons with total angular momentum J relative to the state $J_{A'}\ M_{A'}$. The state $|\eta\ J\ M_J>$ results from coupling two single particle states with angular momenta j_1, j_2 which are abbreviated by the parameter η. By transforming from j - j to L - s coupling we can write

$$|\eta\,(j_1 j_2)\, J M_J> = \tag{4}$$
$$\sum_{L M s_x \sigma_x} \mathcal{G}_{L s_x J}(\eta)\, |l_1 l_2, L M>\, |s_x \sigma_x>\, (L M s_x \sigma_x | J M_J)\,.$$

Here $\mathcal{G}_{L s_x J}(\eta)$ are the usual symmetrized [13] Ls - jj recoupling coefficients and $| s_x \, \sigma_x >$ is a spin-only wave function for the two nucleons with total spin s_x. The dependence on the position coordinates $\mathbf{r}_1$ and $\mathbf{r}_2$ of the two nucleons relative to A (Fig. 1) is contained in $| l_1 \, l_2 \, , \, LM >$.

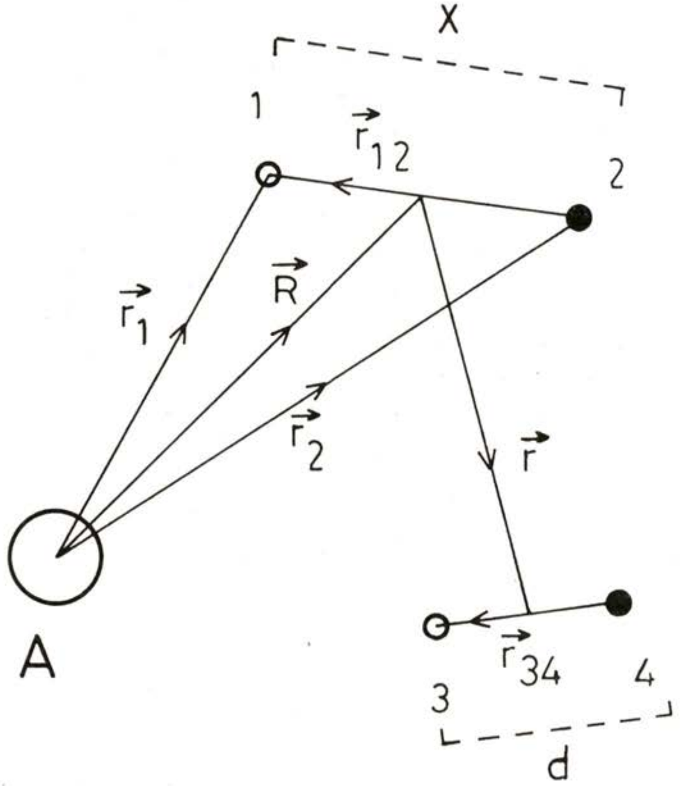

Fig. 1 — Coordinate vectors for a A (α , d) B reaction.

It is now assumed that there is no exchange of nucleons between particles in the entrance and exit channels, no excitation of the target and no reorientation of the target spin through spin-dependent forces. With this assumption the integration over the target internal coordinates selects from eq. (3) the term A′ = A in which the target is in its ground state. Putting eqs. (3) and (4) into eq. (1), performing the integration over the internal coordinates of A and writing the resulting expression in the form of eq. (2) we find that

$$B^{l\lambda}_{sJ} = \sum_{\eta s_x L} \mathcal{S}_J(\eta)\, \mathcal{G}_{L s_x J}(\eta) \sum_{L'} A^{LL'}_{lsJs_x}\, \beta^{l\lambda}_{s_x L'L} . \qquad (5)$$

The coefficients $A^{LL'}_{lsJs_x}$ are the same as in ref. [14] and are given by

$$A^{LL'}_{lsJs_x} = \hat{s}_a\, \hat{l}\, (-1)^{J-s-l-L'}\, W(L s_x l s ; J L') \qquad (6)$$

while

$$\beta^{l\lambda}_{s_x L'L} = \frac{\hat{L}'^2}{\hat{s}_a\, \hat{s}} \sum_{\substack{\sigma_x \sigma_a \sigma_b \\ \sigma M M'}} (-1)^{M'} (L M\, L'\text{-}M' \mid l\lambda)\, (-1)^{s_b-\sigma_b} (s_a \sigma_a s_b \text{-} \sigma_b \mid s\sigma)\, (L' M' s_x \sigma_x \mid s\sigma) \qquad (7)$$
$$\langle b\, s_b\, \sigma_b ; s_x \sigma_x ; l_1 l_2 L M ; \mathbf{k}_b \mid T \mid a\, s_a \sigma_a ; \mathbf{k}_a \rangle .$$

Here $(2s+1)^{1/2}$ is abbreviated by $\hat{s}$. We notice that the total orbital angular momentum transfer in the reaction, l, is composed of a part L and a part L′ which in turn results from the decomposition of the spin transfer s into a spin part s_x and an orbital part L′.

In the microscopic approach to two-nucleon transfer reactions the amplitudes $\beta^{l\lambda}_{s_x L'L}$ are calculated from states $| l_1\, l_2\, LM \rangle$ constructed from shell model wave functions in the nucleon coordinates $\mathbf{r}_1$ and $\mathbf{r}_2$. However to obtain the projectile form factor it is convenient to transform from the coordinates $\mathbf{r}_1$ and $\mathbf{r}_2$ to $\mathbf{r}_{12} = \mathbf{r}_1 - \mathbf{r}_2$ and $\mathbf{R} = (\mathbf{r}_1 + \mathbf{r}_2)/2$. These vectors are represented schematically in Fig. 1. Using a basis of normalized wave functions ϕ_{nl} we can perform the expansion

$$\langle \mathbf{r}_1 \mathbf{r}_2 \mid l_1 l_2 L M \rangle = \sum_{n l_x N\Lambda} c_{n l_x N\Lambda}(\eta)\, [\, \phi_{n l_x}(\mathbf{r}_{12}) \otimes \phi_{N\Lambda}(\mathbf{R})\,]^M_L \qquad (8)$$

where n and N are quantum numbers that specify the number of nodes of the wave functions ϕ. In the particular case of harmonic oscillator wave functions the $c_{n l_x N \Lambda}$ are the well known Moshinsky coefficients [15].

We now assume that the reaction is a one-step process and take V_{bx} for the transfer interaction. It is then straightforward to conclude that the T amplitude in eq. (7) depends on the internal structure of the projectile through the matrix element $< bs_b \sigma_b ; nj_x m_x | V_{bx} | as_a \sigma_a >$. Here $\mathbf{j}_x = \mathbf{l}_x + \mathbf{s}_x$ is the total angular momentum of the transferred two-nucleon cluster.

To proceed with the analysis of the transition matrix elements we use the DWBA theory. No spin dependent interactions either in the entrance or the exit channel are considered in order to simplify the discussion. With this assumption the DWBA amplitude in eq. (7) is [14]

$$
\begin{aligned}
& < b\, s_b\, \sigma_b ; s_x\, \sigma_x ; l_1\, l_2\, L\, M ; \mathbf{k}_b | T | a\, s_a\, \sigma_a ; \mathbf{k}_a > = \\
& \sum_{\substack{nN\Lambda\xi \\ l_x \lambda_x j_x m_x}} c_{nl_x N\Lambda}(\eta)\, (l_x \lambda_x \Lambda\, \xi | L\, M)\, (l_x \lambda_x s_x \sigma_x | j_x m_x) \\
& \qquad \int d^3R \int d^3r\, \chi_b^{(-)*} (\mathbf{k}_b , \mathbf{r}_b)\, \phi_{N\Lambda} (R)\, Y_\Lambda^{\xi *} (\hat{\mathbf{R}}) \\
& \qquad < b\, s_b \sigma_b ; x (nl_x s_x)\, j_x m_x | V_{bx} | a\, s_a \sigma_a > \chi_a^{(+)} (\mathbf{k}_a , \mathbf{r}_a) .
\end{aligned}
\tag{9}
$$

Here χ_a and χ_b are distorted waves and $\mathbf{r}$ is the displacement vector between the centers of mass of the two-nucleon clusters x and b .

3 — (α , d) AND (d , α) REACTIONS

Our present interest is to consider the particular case of (α , d) reactions. The range of n , l_x , s_x values to be considered in eqs. (5), (7) and (9) depends on the assumptions that are made regarding the wave functions of the α - particle and residual nucleus. Conservation of isospin implies that the transferred two-nucleon cluster has $T = 0$. Thus it must be either an even parity state with $s_x = 1$ or an odd parity state with $s_x = 0$. The contribution from the latter type of state is believed to be small since it can only arise from the overlap with odd parity components in the variable $\mathbf{r}_{12}$ in the α particle.

It is therefore usually assumed that the transferred two-nucleon cluster has even parity and only the $l_x = 0$ state is taken into account in DWBA calculations. Furthermore it is frequently supposed that the two nucleons are in a relative S state with no nodes (n = 0). However we note that the $l_x = 2$ states have a non-vanishing overlap with parts of the α particle wave function and in particular with its D-state component.

With $l_x = 0$ we conclude that $j_x = 1$ and the V_{dx} matrix element of eq (9) can be expanded as [7]

$$\begin{aligned} &< d\ 1\ \sigma_d\ ;\ x\,(n\,0\,1)\ 1\ \sigma_x \,|\, V_{dx} \,|\, \alpha > = \\ &\quad 1/2 \sum_{L'=0,2} (-1)^{\sigma_d} (L' M' 1\, \sigma_x \,|\, 1 - \sigma_d)\, v_{nL'}(r)\, Y_{L'}^{M'}(\hat{r})\ . \end{aligned} \tag{10}$$

The vector $\mathbf{r}$ represented in Fig. 1 is the separation between the centers of mass of the clusters; $\mathbf{r} = (\mathbf{r}_{32} + \mathbf{r}_{41})\,/\,2$ with $\mathbf{r}_{ij} = \mathbf{r}_i - \mathbf{r}_j$. As before we denote by 1,2 the transferred nucleons and by 1,3 and 2,4 the identical particles in the α particle. Conservation of parity implies that L′ can only be 0 and 2. The L′ = 0 and L′ = 2 terms on the right hand side of eq. (10) correspond to two different spin configurations in the α particle in which the spins of the two spin one clusters are antiparallel and parallel, respectively. When substituting eqs. (9) and (10) into eq. (7) and performing the summations over magnetic quantum numbers it is found that the orbital angular momentum L′ in eq. (10) is in fact the same as L′ in eq. (7). This gives

$$\beta^{l\lambda}_{1L'L} = \sum_{nN} c_{n0NL}(\eta)\ \tilde{B}^{l\lambda}_{nNLL'}\ , \tag{11}$$

with

$$\begin{aligned} \tilde{B}^{l\lambda}_{nNLL'} = \sqrt{3}/2 &\sum_{MM'} (-1)^{1+M'} (L\,M\,L' {-M'} \,|\, l\,\lambda) \\ &\int d^3R \int d^3r\ \chi_d^{(-)^*}(\mathbf{k}_d, \mathbf{r}_d)\ \phi_{NL}(R)\ Y_L^{M^*}(\hat{\mathbf{R}}) \\ &\quad v_{nL'}(r)\, Y_{L'}^{M'}(\hat{r})\, \chi_\alpha^{(+)}(\mathbf{k}_\alpha, \mathbf{r}_\alpha)\ . \end{aligned} \tag{12}$$

Using eqs. (5) and (11) we can write

$$B^{l\lambda}_{1J} = \sum_{nNL} G_{nNLJ} \sum_{L'} A^{LL'}_{l1J1}\ \tilde{B}^{l\lambda}_{nNLL'}\ . \tag{13}$$

Here the information on the nuclear structure of the A + 2 nucleus is as much as possible concentrated in the amplitude

$$G_{nNLJ} = \sum_{\eta} \mathcal{S}_J(\eta)\ \mathcal{G}_{L1J}(\eta)\ c_{n0NL}(\eta)\ . \tag{14}$$

On the other hand the information on the α particle is contained in the sum over L′.

The differential cross section for the A (α , d) B reaction is an incoherent sum over l and J

$$\begin{aligned} d\sigma / d\Omega \ \propto\ & \sum_{Jl\lambda} (2J_B + 1) / (2l + 1)\ |B^{l\lambda}_{1J}|^2 \\ = & \sum_{Jl\lambda} (2J_B + 1) / (2l + 1)\ \Big|\sum_{nNL} G_{nNLJ} \sum_{L'} A^{LL'}_{l1J1}\ \tilde{B}^{l\lambda}_{nNLL'}\Big|^2 \ . \end{aligned} \tag{15}$$

With the inclusion of the α - particle D - state the total orbital angular momentum transfer l may not be equal to L . Furthermore we notice that the $L' = 2$ contribution introduces a J dependence into the cross section through the $A^{LL'}_{l1J1}$ coefficients.

Here we are particularly interested in the analysing powers of the inverse reaction B ($\vec{d}$, α) A . From invariance under time reversal the analysing powers T_{kq} of the B ($\vec{d}$, α) A reaction are related with the polarization tensors t_{kq} of the A (α , $\vec{d}$) B reaction by [11]

$$T_{kq} = (-1)^k\ t_{kq} \tag{16}$$

when using the same coordinate system on both sides of eq. (16). The polarization tensors t_{kq} are given by

$$t_{kq} = \mathrm{Trace}\,(T^{\dagger}\,\tau_{kq}(1)\,T) \,/\, \mathrm{Trace}\,(T^{\dagger}\,T) \tag{17}$$

where T is the transition amplitude for the (α, d) reaction and $\tau_{kq}(1)$ are the usual spin one operators [16]. Using eqs. (2), (16) and (17) we obtain

$$\begin{aligned} T_{kq} = & -\sqrt{3}\ \Big(\sum_{l\lambda} (2l + 1)^{-1}\, |B^{l\lambda}_{1J}|^2\Big)^{-1} \\ & \sum_{Jl\lambda l'\lambda'} (-1)^{k+J+\lambda}\ W(1\,l\,1\,l'; Jk)\ (l\,{-\lambda}\ l'\,\lambda' \,|\, kq)\ B^{l\lambda}_{1J}\ B^{l'\lambda'^{*}}_{1J} . \end{aligned} \tag{18}$$

Unlike the cross section the T_{kq} involve a coherent sum over $B^{l\lambda}_{1J}$ amplitudes with different l .

4 — THE ASYMPTOTIC D- to S-STATE RATIO IN THE α PARTICLE

A full finite range DWBA calculation for $B(\vec{d}, \alpha)A$ reactions requires the knowledge of the radial wave functions $v_{nL'}(r)$, defined in eq. (10). We consider only the V_{dx} matrix element for $n = 0$ because the dominant component of the expansion (8) in the internal variable $\mathbf{r}_{12}$ of the transferred cluster is an S state with no nodes [17]. In the following it is therefore assumed that $n = 0$ and all dependence on n is dropped. However we note that at least in the $L' = 0$ part of the transition amplitude the contributions from S state cluster states with $n \neq 0$ are not negligible for some cases [18].

The overlap between the α particle wave function and the two spin-one clusters has an expansion analogous to eq. (10) [5, 7]

$$< \phi_d^{\sigma_d}(3,4)\, \phi_x^{\sigma_x}(1,2) \mid \phi_\alpha > = \tag{19}$$
$$1/2 \sum_{L'=0,2} (-1)^{\sigma_d} (L' M' 1 \sigma_x \mid 1\, {-\sigma_d})\; u_{L'}(r)\; Y_{L'}^{M'}(\hat{r})\,.$$

This function satisfies the equation

$$-(B_\alpha - B_d - B_x + T_r) < \phi_d^{\sigma_d}(3,4)\, \phi_x^{\sigma_x}(1,2) \mid \phi_\alpha > \tag{20}$$
$$= < \phi_d^{\sigma_d}(3,4)\, \phi_x^{\sigma_x}(1,2) \mid V_{dx} \mid \phi_\alpha >$$

where on the right hand side the matrix element is the same as in eq. (10). B_α, B_d, B_x are binding energies and T_r is the kinetic energy in $\mathbf{r}$. Combining eqs. (10), (19) and (20) we conclude that the radial wave functions $u_{L'}$ and $v_{L'}$ are related by

$$v_{L'}(r)\; Y_{L'}^{M'}(\hat{r}) = -(\hbar^2/2M)\,(\alpha^2 - \nabla^2)\, u_{L'}(r)\; Y_{L'}^{M'}(\hat{r})\,, \tag{21}$$

where $\alpha = [\,2M(B_\alpha - B_d - B_x)/\hbar^2\,]^{1/2}$ is the wave number of the relative motion between clusters in the α particle. Eq. (21) shows that asymptotically, for large r,

$$u_{L'}(r) \xrightarrow[r \to \infty]{} \mathcal{N}_{L'}\; i^{L'}\; h_{L'}(i\,\alpha\, r)\,, \tag{22}$$

neglecting the Coulomb interaction between clusters. The asymptotic D- to S-state ratio in the α particle is [5]

$$\rho = \mathcal{N}_2 / \mathcal{N}_0 \ . \tag{23}$$

In low energy (d, α) reactions the DWBA calculations are not very sensitive to the precise and presently unknown short range behaviour of the functions $u_{L'}(r)$ [7, 8]. The calculated tensor analyzing powers depend to a good approximation upon u_0 and u_2 only through the parameter D_2 defined by [1]

$$D_2 = \int_0^\infty u_2(r)\, r^4\, dr \,/\, 15 \int_0^\infty u_0(r)\, r^2\, dr \tag{24}$$

An alternative expression

$$D_2 = (2M / \hbar^2 \alpha^2) \int_0^\infty v_2(r)\, r^4\, dr \,/ \int_0^\infty u_0(r)\, r^2\, dr \tag{25}$$

is obtained using eq. (21) to relate the coefficients of the k^2 term in a power series expansion of u_2 and v_2 in momentum space. The substitution of the asymptotic forms (22) into eq. (24) gives the well known relation [1, 19]

$$D_2 \simeq \rho / \alpha^2 \, . \tag{26}$$

However the reliability of this approximate relation is expected to be much smaller in (d, α) reactions than in (d, p) reactions because of the large α particle binding energy.

A non-vanishing D_2 can only be obtained through the nucleon-nucleon tensor interaction in the four body bound system. To obtain an estimate of D_2 we assume, in analogy with what is presently known about the three body bound system [3], that u_0 and u_2 are primarily determined, respectively, by the overlaps $< \phi_d(3,4)\ \phi_x(1,2) \mid \phi_{\alpha S} >$ and $< \phi_d(3,4)\ \phi_x(1,2) \mid \phi_{\alpha D} >$ with the S and D state components of the α particle wave function

$$\phi_\alpha = \phi_{\alpha S} + \phi_{\alpha D} \ . \tag{27}$$

It is important to emphasize that this is an approximation. For instance it is easily verified that the S state component $\phi_{\alpha S}$ gives contributions to u_2 through the D-states in the spin one clusters. These contributions are probably small because they arise from low probability components in $\phi_{\alpha S}$ that result from coupling states with non-zero orbital angular momenta in the coordinates $\mathbf{r}_{12}$, $\mathbf{r}_{34}$, $\mathbf{r}$ to a total $L' = 0$.

A model to generate $\phi_{\alpha D}$ is required in order to calculate D_2 . Using a perturbative treatment [5] we can write, to first order in the tensor interaction,

$$(T + \sum_{i<j} V_c (i , j) + B_\alpha) \mid \phi_{\alpha D} > \simeq - \sum_{i<j} V_T (i , j) \mid \phi_{\alpha S} > . \qquad (28)$$

Here $V_c (i , j)$ and

$$V_T (i , j) = V_T (r_{ij}) S_{12} (i , j) \qquad (29)$$

are the central and tensor parts of the nucleon-nucleon interaction. The overlap of eq. (28) with the spin one clusters satisfies the equation

$$\begin{aligned} (B_\alpha - B_d - B_x + T_r) < \phi_d^{\sigma_d} (3 , 4) \phi_x^{\sigma_x} (1 , 2) \mid \phi_{\alpha D} > \simeq \\ - < \phi_d^{\sigma_d} (3 , 4) \phi_x^{\sigma_x} (1 , 2) \mid \sum_{i < j} V_T (i , j) \mid \phi_{\alpha S} > \end{aligned} \qquad (30)$$

if the central interactions between clusters are neglected. This approximation is based on the fact that the effect of V_c is reduced by the centrifugal barrier associated with the D-state in $\mathbf{r}$.

On the right hand side of eq. (30) there are no contributions from $V_T (1,3)$ and $V_T (2,4)$ since the nucleon pairs 1,3 and 2,4 are in singlet states. Furthermore the tensor interactions $V_T (1,2)$ and $V_T (3,4)$ do not generate a relative D-state motion of the cluster if we consider only the dominant component of $\phi_{\alpha S}$ exclusively with S states in $\mathbf{r}_{12}$, $\mathbf{r}_{34}$, $\mathbf{r}$. Thus combining eqs. (10), (21) and (30) yields

$$\begin{aligned} < \phi_d^{\sigma_d} (3 , 4) \phi_x^{\sigma_x} (1 , 2) \mid V_T (2 , 3) + V_T (1 , 4) \mid \phi_{\alpha S} > = \\ 1/2 (-1)^{\sigma_d} (2 M' 1 \sigma_x \mid 1 -\sigma_d) v_2 (r) Y_2^{M'} (\hat{r}) . \end{aligned} \qquad (31)$$

Using eqs. (19), (25) and (31) it is now straightforward to calculate the parameter D_2. This calculation is considerably simplified by the use of gaussian wave functions to represent the bound states

$$\phi_{\alpha S} = E(\lambda) \exp[-\lambda(r_{12}^2 + r_{34}^2 + 2 r^2)/4] \chi_0(1,3) \chi_0(2,4), \quad (32)$$

$$\phi_d^{\sigma_d}(3,4) \phi_x^{\sigma_x}(1,2) = F^2(\nu) \exp[-\nu(r_{34}^2 + r_{12}^2)/2] \chi_1^{\sigma_d}(3,4) \chi_1^{\sigma_x}(1,2). \quad (33)$$

In eq. (33) we made the usual assumption of describing x by a deuteron wave function. $E(\lambda) = 2^{-3/2}(\lambda/\pi)^{9/4}$ and $F(\nu) = (\nu/\pi)^{3/4}$ are normalization constants and $\chi_0(i,j)$ and $\chi_1^{\sigma}(i,j)$ are singlet and triplet spin wave functions. The parameters λ and ν are related to the α - particle and deuteron rms radius by

$$< r^2 >^{1/2}_{\alpha \text{ particle}} = 3/(2\sqrt{2\lambda}), \quad (34)$$

$$< r^2 >^{1/2}_{\text{deuteron}} = 1/2\sqrt{3/2\nu}. \quad (35)$$

With the wave functions (32) and (33), the radial function u_0 is a gaussian function

$$u_0(r) = 4\sqrt{2}\,\delta^{-3}(\pi^{-1}\lambda^9\nu^6)^{1/4} e^{-\lambda r^2/2} \quad (36)$$

where $\delta = \nu + \lambda/2$. To calculate $v_2(r)$ from eq. (31) it is convenient to write

$$\chi_0(1,3)\chi_0(2,4) = -1/2[\chi_0(1,4)\chi_0(3,2) + \sum_m (-1)^{1+m} \chi_1^m(1,4)\chi_1^{-m}(3,2)] \quad (37)$$

since we are interested in the tensor force in the nucleon pairs 1,4 and 2,3. Using the relation

$$S_{12}(\hat{r})\chi_1^{\sigma} = 4\sqrt{2\pi} \sum_{\sigma' M} (1\,\sigma'\,2\,M \,|\, 1\,\sigma) Y_2^M(\hat{r}) \chi_1^{\sigma'} \quad (38)$$

and eqs. (32) and (33) we obtain

$$v_2(r) = 2^7 (\lambda^3/\pi)^{3/4} (\nu/\delta)^{3/2} \exp[-(\nu+\lambda) r^2] \int_0^\infty j_2(2 i \delta r x) \exp(-\delta x^2) V_T(x) x^2 dx . \tag{39}$$

Finally doing the integrations over r in eq. (25) gives

$$D_2 = (8/15)(B_\alpha - 2B_d)^{-1} (\lambda^3/\pi)^{1/2} [\delta/(\nu+\lambda)]^{7/2} \int_0^\infty V_T(x) \exp[-\lambda\delta x^2/2(\nu+\lambda)] x^4 dx . \tag{40}$$

Using the one-pion-exchange tensor potentital (OPEP)

$$V_T(r) = -C_T h_2(i\mu r) \tag{41}$$

with $C_T = 10.463$ MeV and $\mu = 0.7$ fm^{-1} [20] we obtain $D_2 = -0.153$ fm^2 for deuteron and α particle rms radius of 1.96 fm and 1.42 fm [21], respectively. The introduction of a cutoff factor [22], $1 - \exp(-Ar^2)$ where $A = 0.735$ fm^{-2}, in the OPEP tensor potential increases D_2 to -0.117 fm^2. This change of 23 % indicates that the parameter D_2 depends on the behaviour of the tensor interaction at distances smaller than 2 fm. The sensitivity of D_2 to the tensor interaction at short distances is much stronger in (d,α) than in (d,p), (d,t) or $(d,{}^3He)$ reactions. The values of D_2 become slightly larger when either the rms radius of the deuteron or the rms radius of the α particle are increased. For instance $D_2 = -0.124$ fm^2 for deuteron and α particle rms radii of 2.10 fm and 1.70 fm, respectively.

Although the model used to calculate D_2 is probably realistic the bound state wave functions are not adequate. In fact D_2 is very sensitive to the asymptotic region of large r . Thus we can expect that the calculated values of D_2 are overestimated because they were obtained with gaussian functions. The same problem of overestimated values of D_2 was also found in calculations of D_2 for ${}^3H$ when using wave functions with incorrect asymptotic behaviour [23]. Calculations based on the very simplified model for ρ developed in ref. [5] give $-0.35 < D_2 < -0.15$ fm² [24]. This model has the unrealistic feature that the tensor interaction between clusters depends only on the coordinate **r** but, on the other hand, the calculations were performed with wave functions $u_0(r)$ with correct asymptotic behaviour.

5 — PERIPHERAL MODEL OF (d , α) AND (α , d) REACTIONS

To study the dependence of the cross section and of the analysing powers on the amplitudes G_{NLJ} and also on the asymptotic D- to S-state ratio ρ we use the peripheral model developed in refs. [5, 25]. The bound state wave functions of the transferred two nucleon cluster in the α particle and in the nucleus B are represented by their asymptotic forms for large r

$$u_{L'}(r) \simeq \mathcal{N}_{L'}\, i^{L'}\, h_{L'}(i\alpha r)\,, \tag{42}$$

$$\phi_{NL}(r) \simeq \mathcal{N}_{NL}\, i^{L}\, h_{L}(i\beta r)\,. \tag{43}$$

Here β is the wave number corresponding to the binding energy of the cluster x in B and $\mathcal{N}_{NL}$ are asymptotic normalization constants. For small recoil effects the $\tilde{B}^{l\lambda}_{NLL'}$ amplitudes can be approximated by

$$\tilde{B}^{l\lambda}_{NLL'} \simeq \sqrt{3}/2 \sum_{MM'} (-1)^{1+M'} (L\,M\,L' -M' \mid l\,\lambda)$$
$$\int d^3R \int d^3r\; \chi^{(-)^*}_{d}(\mathbf{k}_d, (m_A/m_B)\,\mathbf{R})\, \phi_{NL}(|\mathbf{R} - a\,\mathbf{r}|) \tag{44}$$
$$Y^{M^*}_{L}(\widehat{\mathbf{R} - a\,\mathbf{r}})\, v_{L'}(r)\, Y^{M'}_{L'}(\hat{\mathbf{r}})\, \chi^{(+)}_{\alpha}(\mathbf{k}_\alpha, \mathbf{R})\,.$$

The value of the parameter a depends on the particular assumptions made in the derivation of eq. (44). For instance if we choose **R** as the average of the arguments of the two distorted waves [26] then $a = 3/4$. In the usual form of the non-recoil approximation [27] for heavy ion transfer reactions $a = 1$.

With the bound state wave functions (42) and (43) the **r** integration in eq. (44) can be performed analytically. In fact the formula A. 46 of ref. [14] gives

$$\int d^3r\, i^L h_L (i\,\beta\, | \mathbf{R} - \mathbf{r} |)\, Y_L^{M^*} (\hat{\mathbf{R} - \mathbf{r}})\, (\nabla^2 - \alpha^2) i^{L'} h_{L'} (i\, \alpha\, r)\, Y_{L'}^{M'} (\hat{\mathbf{r}})$$
$$= \sqrt{4\,\pi}\, \hat{L}' (L'0\, l\, 0 \,|\, L0) (-1)^{L'+M'} (L\, M\, L' - M' \,|\, l\, \lambda) \qquad (45)$$
$$(\beta^{L'} / \alpha^{L'+1})\, i^l h_l (i\, \beta\, R)\, Y_l^{\lambda^*} (\hat{\mathbf{R}}) .$$

Therefore using eqs. (42), (43) and (45) we obtain

$$\sum_{MM'} (-1)^{1+M'} (L\, M\, L' - M' \,|\, l\, \lambda) \cdot$$
$$\int d^3r\, \phi_{NL} (|\mathbf{R} - a\, \mathbf{r}|)\, Y_L^{M*} (\hat{\mathbf{R} - a\, \mathbf{r}})\, v_{L'} (r)\, Y_{L'}^{M'} (\hat{\mathbf{r}}) = \qquad (46)$$
$$- \frac{\hbar^2}{2\, M\alpha} \mathcal{N}_{NL}\, \mathcal{N}_{L'}\, \sqrt{4\,\pi}\, \hat{L}' \, (L'0\, l\, 0 \,|\, L0) \left(\frac{a\, \beta}{\alpha} \right)^{L'} i^l h_l (i\, \beta\, R)\, Y_l^{\lambda^*} (\hat{\mathbf{R}})$$

The neglect of the recoil induced by the transfer implies that only normal parity values of l are allowed

$$l + L + L' = \text{even} . \qquad (47)$$

The substitution of eq. (46) into eq. (44) and the use of plane waves to represent the scattering states gives

$$\tilde{B}_{NLL'}^{l\lambda} = I_l (Q)\, Y_l^{\lambda^*} (\hat{Q})\, \hat{L}' (L'0\, l\, 0 \,|\, L0)\, (a\, \beta / \alpha)^{L'}\, \mathcal{N}_{NL}\, \mathcal{N}_{L'} . \quad (48)$$

Here

$$\mathbf{Q} = \mathbf{k}_\alpha - (m_A / m_B)\, \mathbf{k}_d \ , \qquad (49)$$

is the momentum transfer in the reaction and

$$I_l (Q) = 2\, \sqrt{3}\, \pi\, (\hbar^2 / M\alpha)\, (-1)^{l+1} \int h_l (i\, \beta\, R)\, j_l (Q\, R)\, R^2 d\, R . \qquad (50)$$

Finally the combination of eqs. (13) and (48) yield

$$B_{1J}^{l\lambda} = U_{lJ} Y_l^{\lambda^*} (\hat{Q}) \ , \tag{51}$$

with

$$U_{lJ} = I_l (Q) \sum_{LL'} S_{LJ} \hat{L}' (L' 0 \, l \, 0 \,|\, L0) \, A_{l1J1}^{LL'} \, \mathcal{N}_{L'} (a \, \beta \, / \, \alpha)^{L'} \ . \tag{52}$$

The information on the A + 2 nucleus is now entirely contained in the spectroscopic amplitude

$$S_{LJ} = \sum_{N} G_{NLJ} \mathcal{N}_{NL} = \sum_{N\eta} \mathcal{S}_J (\eta) \, \mathcal{G}_{L1J} (\eta) \, c_{0NL} (\eta) \, \mathcal{N}_{NL} \ . \tag{53}$$

Using eqs. (15) and (51) it is easily concluded that the cross section is an incoherent sum of the square of the amplitudes U_{lJ} over l and J

$$d\sigma \, / \, d\Omega \ \propto \ (2J_B + 1) \, / \, 4\pi \sum_{lJ} U_{lJ}^2 \ . \tag{54}$$

It is also straightforward to obtain an expression for the analysing powers T_{kq} as a function of U_{lJ}. Since the dependence on the magnetic quantum number in $B_{1J}^{l\lambda}$ is now given by the spherical harmonic Y_l^λ the summation over λ and λ' in eq. (18) gives rise to a Clebsch-Gordan coefficient $(l \, 0 \, l' 0 \,|\, k0)$ and implies that the T_{kq} are proportional to $Y_k^q (\hat{Q})$. Furthermore there is a restriction in the values of k. In a given transition the allowed values of L have all the same parity and L′ is even. Therefore the selection rule (47) implies that all values of the total orbital angular momentum transfer l have the same parity. In conclusion the analyzing powers with k odd vanish in the peripheral model. This is a general property of plane wave approximations [28]. For k = 2 eqs. (18) and (51) yield

$$T_{2q} = - (8 \, \pi \, / \, 5)^{1/2} \, A \, Y_2^q (\hat{Q}) \ , \tag{55}$$

with

$$A = (3/2)^{1/2} (\sum_{lJ} U_{lJ}^2)^{-1} \sum_{ll'J} \hat{l} \, \hat{l}' \, (l \, 0 \, l' 0 \,|\, 20) \ W (l \, 1 \, l' 1 \, ; J2) \ U_{lJ} \, U_{l'J} \ . \tag{56}$$

Eq. (55) shows that in the peripheral model the angular dependence of the tensor analyzing powers is essentially determined by the spherical harmonics $Y_2^q(\hat{Q})$. In the Madison convention coordinate system [16] where the z axis is along $\mathbf{k}_d$ and the y axis is along $\mathbf{k}_d \times \mathbf{k}_p$

$$T_{20} = -(1/\sqrt{2})\, A\, (3 \cos^2 \gamma - 1)\,, \tag{57a}$$

$$T_{21} = \sqrt{3}\, A \sin \gamma \cos \gamma\,, \tag{57b}$$

$$T_{22} = -(\sqrt{3}/2)\, A \sin^2 \gamma\,. \tag{57c}$$

The angle

$$\gamma = \text{arc tg} \{ \sin \Theta\, [\cos \Theta - (m_A / m_B)\, (k_d / k_\alpha)\,]^{-1} \} \tag{58}$$

is the angle between $\mathbf{Q}$ and $\mathbf{k}_d$ and Θ is the scattering angle. The relations (57) acquire a particularly simple form when the tensor analyzing powers are expressed in a cartesian representation

$$A_{xx} = -(1/\sqrt{2})\, (T_{20} - \sqrt{6}\, T_{22}) = (A/2)\, (3 \cos 2\gamma - 1)\,, \tag{59a}$$

$$A_{yy} = -(1/\sqrt{2})\, (T_{20} + \sqrt{6}\, T_{22}) = A\,, \tag{59b}$$

$$A_{zz} = -(A_{xx} + A_{yy}) = -(A/2)\, (3 \cos 2\gamma + 1)\,. \tag{59c}$$

The most significant aspect of eq. (59) is that A_{yy} is, to a good approximation, independent of Θ. This property of A_{yy} is common to other reactions [25] and has a simple physical interpretation. The difference between the unpolarized cross section and a cross section for a spin orientation perpendicular to the reaction plane is insensitive to the scattering angle because the correlation between spin and deformation implies that the wave function of relative motion between clusters has spherical symmetry in the reaction plane. This spherical symmetry is broken for other spin orientations and as a result the tensor analyzing powers become dependent on Θ. For instance the analyzing power A_{xx} has a minimum of -2A at $\Theta = \text{arc cos}\, (m_A k_d / m_B k_\alpha)$ and is equal to A at $\Theta = 0°$ and $180°$.

5.1 — *Natural parity transitions*

In natural parity transitions $L = J$. From eqs. (6) and (52) and with the help of tables of angular momentum coupling coefficients [12] we obtain

$$U_{lJ} = \delta_{lJ} (\mathcal{N}_0 / \sqrt{3}) I_J S_{JJ} [1 + (\rho / \sqrt{2}) (a \beta / \alpha)^2] . \tag{60}$$

The differential cross section in a transition with a given J is

$$(d\sigma / d\Omega)_J \propto (\mathcal{N}_0^2 / 12\pi) \{ I_J S_{JJ} [1 + (\rho / \sqrt{2}) (a \beta / \alpha)^2] \}^2 . \tag{61}$$

The fact that ρ is negative implies that the D-state of the α particle decreases the cross section of (d , α) and (α , d) natural parity transitions. This effect is particularly noticeable in transitions with large β.

For the tensor analyzing powers the substitution of eq. (60) into eq. (56) gives $A = - 1/2$ and therefore

$$A_{yy} = - 1/2 . \tag{62}$$

This simple result is interesting to understand. A_{yy} is equal to the polarization component [16]

$$p_{yy} = < 3 s_y^2 - 2 > \tag{63}$$

of the outgoing deuteron beam in a $(\alpha , \vec{d})$ reaction. In a peripheral reaction the vector $\mathbf{L}$ is perpendicular to the reaction plane and therefore either parallel or antiparallel to the y axis. For $L = J$ and because $\mathbf{J} = \mathbf{L} + \mathbf{s}_x$, the spin $\mathbf{s}_x$ is either parallel or antiparallel to the z axis. This is also true for the outgoing deuteron because of the spin correlation between the spin one clusters in the α particle. Thus in natural parity transitions the (α , d) reaction acts as a spin filter supressing the $m_z = 0$ states. In a polarization state where $m_z = \pm 1$, $< s_y^2 > = 1/2$ and therefore from eq. (63) $p_{yy} = A_{yy} = - 1/2$.

5.2 — *Unnatural parity transitions*

In unnatural parity transitions for a fixed J the orbital angular momentum of the transferred cluster can be $L = J - 1$ and $L = J + 1$. Again from eqs. (6) and (52) we obtain [12] for $L = J - 1$

$$U_{J-1,J} = (\mathcal{N}_0/\sqrt{3})\, I_{J-1} [S_{J-1,J} + (\rho/\sqrt{2})(2J+1)^{-1}(a\beta/\alpha)^2 \\ (3[J(J+1)]^{1/2} S_{J+1,J} - (J-1) S_{J-1,J})] \tag{64}$$

and for $L = J + 1$

$$U_{J+1,J} = (\mathcal{N}_0/\sqrt{3})\, I_{J+1} [S_{J+1,J} + (\rho/\sqrt{2})(2J+1)^{-1}(a\beta/\alpha)^2 \\ (3[J(J+1)]^{1/2} S_{J-1,J} - (J+2) S_{J+1,J})] . \tag{65}$$

Given J , the differential cross section is

$$(d\sigma/d\Omega)_J \propto (1/4\pi)(U^2_{J-1,J} + U^2_{J+1,J}) . \tag{66}$$

Notice that for $\rho = 0$

$$(d\sigma/d\Omega)_J \propto (\mathcal{N}_0^2/12\pi)(I^2_{J-1} S^2_{J-1,J} + I^2_{J+1} S^2_{J+1,J}) \tag{67}$$

is insentitive to the sign of the spectroscopic amplitures $S_{L,J}$.

Eqs. (64-66) show that, because ρ is negative, the D-state of the α particle has generally the effect of increasing the cross section of unnatural parity transitions. This is the case, for instance, of a pure $L = J - 1$ transition and also of a pure $L = J + 1$ transition. The opposite effect of the D-state in natural and unnatural parity transitions introduces in the cross section a J-dependence which qualitatively is in agreement with that observed in the $^{208}Pb(\alpha, d)^{210}Bi$ reaction feeding members of the $\{h_{9/2}, g_{9/2}\}$ multiplet [10].

We now consider the tensor analysing powers in unnatural parity transitions. Eqs. (56) and (59b) give

$$A_{yy} = A = \frac{(J+2)x^2 - 6[J(J+1)]^{1/2} x + J - 1}{2(2J+1)(1+x^2)} \tag{68}$$

with

$$x = U_{J+1,J} \, / \, U_{J-1,J} \; . \tag{69}$$

Thus A_{yy} varies from a minimum value of $-1/2$ for $x = [J/(J+1)]^{1/2}$ to a maximum value of 1 for $x = -[(J+1)/J]^{1/2}$.

In the absence of D-state effects $\rho = 0$ and

$$x = K_J \; S_{J+1,J} \, / \, S_{J-1,J} \tag{70}$$

where $K_J = I_{J+1} \, / \, I_{J-1}$ is a positive quantity due to the form of the integrals (50). Eqs. (55) and (68) show that the T_{2q} have a strong dependence on the spectroscopic amplitudes S_{LJ}. Unlike the cross section they depend on the relative sign of $S_{J-1,J}$ and $S_{J+1,J}$. Fig. 2 shows the values of

$$(A_{yy})_J = (J-1) \, / \, [\, 2\,(2J+1)\,] \tag{71}$$

for a pure $L = J-1$ transition ($x = 0$) and

$$(A_{yy})_J = (J+2) \, / \, [\, 2\,(2J+1)\,] \tag{72}$$

for a pure $L = J+1$ transition ($x = \infty$). Since $K_J > 0$, $x > 0$ when $S_{J+1,J}$ and $S_{J-1,J}$ have the same sign and $x < 0$ when $S_{J+1,J}$ and $S_{J-1,J}$ have opposite signs. The quantity x is a double valued function of A_{yy} . $x < 0$ for $(A_{yy})_J > (J+2) \, / \, [2(2J+1)]$, $x > 0$ for $(A_{yy})_J < (J-1) \, / \, [\,2\,(2J+1)\,]$ and x is either positive or negative for $(J-1)/[2(2J+1)] < (A_{yy})_J < (J+2)/[2(2J+1)]$ as shown in Fig. 2.

In the presence of D-state effects $\rho \neq 0$ and for a pure $L = J-1$ transition

$$x = \frac{K_J}{3\,\rho\, b} \; \frac{3J+1-\rho\, b\,(J+2)}{[\,J\,(J+1)\,]^{1/2}} \tag{73}$$

where $b = (a\,\beta\,\alpha)^2 \, / \, \sqrt{2}$. In a pure $L = J+1$ transition

$$x = 3\,\rho\, b\; K_J \; \frac{[\,J\,(J+1)\,]^{1/2}}{2J+1-\rho\, b\,(J-1)} \; . \tag{74}$$

In both cases $x < 0$. Therefore the effect of the D-state is to increase A_{yy} relative to the values given by eqs. (71) and (72). The substitution of eqs. (73) and (74) into eq. (68) shows that the α particle D-state effect is relatively larger in $L = J - 1$ than in $L = J + 1$ transitions. This result is important to select transitions where the extraction of ρ from T_{2q} experimental data is favoured.

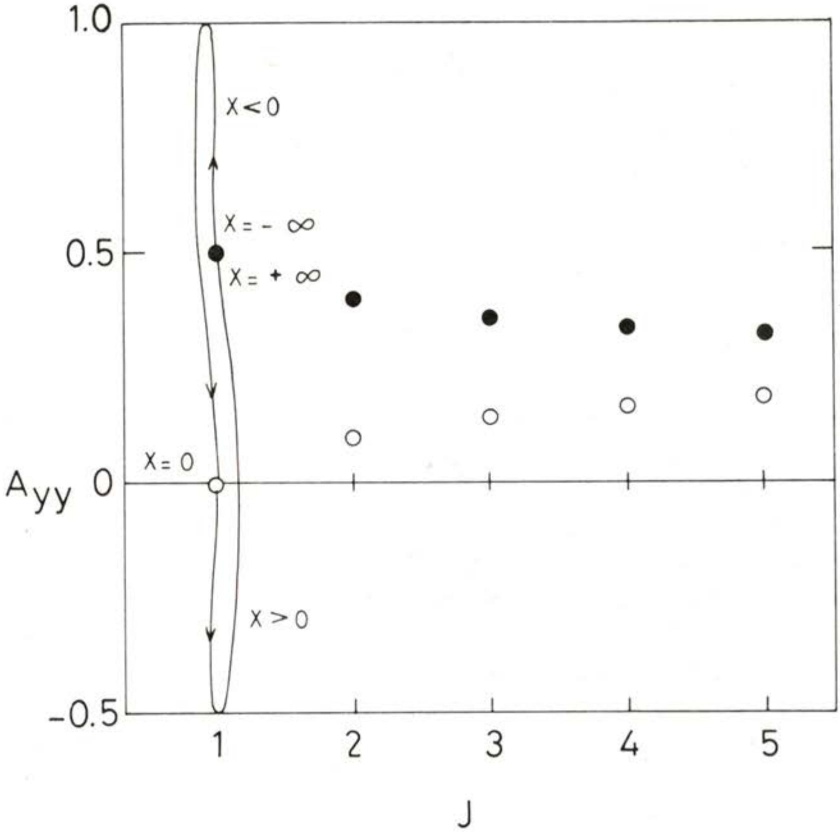

Fig. 2 — The tensor analyzing power A_{yy} of (**d** , α) reactions to unnatural parity states as a function of the total angular momentum transfer J. The open and full points correspond to pure $L = J - 1$ and pure $L = J + 1$ transitions, respectively. For each J , A_{yy} is given by eq. (68) and varies with x from - 1/2 to 1. For $J = 1$ we have represented in a loop the values taken by A_{yy} as function of x .

In an unnatural parity transition with only one pair of values for J , L the measurement of the T_{2q} yields a unique value for x that can be used to estimate ρ. Knowing ρ it becomes possible to determine the amplitudes $S_{J+1,J}$ and $S_{J-1,J}$ in transitions with L mixing. These amplitudes can then be compared with those obtained from shell model calculations.

6 — CONCLUSIONS

A general discussion of the angular momentum structure of the transition amplitude in (α , d) and (d , α) reactions is presented. Particular emphasis is given to the analysis of contributions from the D-state components of the α particle wave function. The parameter D_2 is estimated using a perturbative treatment to first order in the tensor interaction and gaussian wave functions to represent the deuteron and α - particle bound state wave functions. These calculations show that D_2 in (d , α) reactions is sensitive to the form of the nucleon-nucleon tensor interaction at distances smaller than 2 fm. Further calculations of D_2 using more realistic wave functions with correct asymptotic behaviour are required.

The dependence of the cross section and of the tensor analysing powers on the asymptotic D- to S-state ratio ρ and on the spectroscopic amplitudes S_{LJ} is discussed using a plane wave peripheral model. The tensor analyzing power A_{yy} is particularly interesting because it is independent of angle and its value is a simple function of ρ and S_{LJ} . The present analysis indicates that the determination of ρ from T_{2q} data is specially favoured in unnatural parity transitions involving only the orbital angular momentum $L = J - 1$. These occur in (d , α) reactions on closed shell target nuclei leading to outstretched nuclear configurations with $J = L + 1$.

With the peripheral model it is possible to identify the main features of nuclear structure and D-state effects in the cross section and T_{2q} . However the model cannot be applied to the description of iT_{11} and furthermore it cannot be used in a quantitative analysis of the data. For instance the experimental A_{yy} angular distributions oscillate around a certain mean value [5] that varies from transition to transition. This mean value can be interpreted with the peripheral model but to reproduce the oscillatory behaviour it is necessary to perform a DWBA calculation including a spin-orbit interaction in the deuteron channel [7, 8]. An analysis of recent T_{2q} data in (d α) reactions with full finite range DWBA calculations is in progress and shall be presented elsewhere.

We are thankful to W.J. Thompson for illuminating discussions. One of us (FDS) would like to thank the hospitality of the Department of Physics and Astronomy of the University of North Carolina where part of this work was performed.

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