EFFECTS OF THE MAGNETIC FIELD SHAPE IN THE CHARACTERISTICS OF A DOUBLE FOCUSING ELECTRON SPIN POLARIMETER

P. AMORIM (1) e J. P. RIBEIRO (2)

Complexo II do I.N.I.C. — Av. Prof. Gama Pinto, 2 — 1699 Lisboa Codex, Portugal

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ABSTRACT — In this paper we discuss the influence of a magnetic field of the form $B_z = B_0 (R/r)^{1+\alpha}$ on the characteristics of a double focusing $\pi/2$ sector field polarimeter. The influence of α in the radial and axial focusing distances is studied for the energy range 100-1000 keV. We conclude that for $\alpha \neq 0$ the anastigmatism of the polarimeter can be very important for measurements at several hundred keV whereas it is negligible for electrons of a few keV. The effect introduces a systematic error which must be carefully estimated in each experimental situation.

1 — INTRODUCTION

The experimental results of longitudinal polarization of electrons emitted in beta decay have been obtained using different methods [1]. One of them, the Mott scattering method, is an exclusively transverse polarization sensitive method [2] and, therefore, it is necessary to put the electron in a system which converts longitudinal into transverse polarization for any electron energy. This system is usually known as "polarimeter" and can be achieved with a crossed electric and magnetic field configuration which, in the neighbourhood of an equilibrium orbit of radius of curvature R, has the approximate form [3]

$$E_r = E_0 (R/r)^2 , \; E_\theta = E_z = 0 \; ; \; B_r = B_\theta = 0 \; , \; B_z = B_0 (R/r) \qquad (1.1)$$

(1) Centro de Física dos Fenómenos de Ionização Interna da Universidade de Lisboa.

(2) Centro de Física Nuclear da Universidade de Lisboa.

Such an instrument (Fig. 1) has i) independent selection of both electron energy and spin orientation at a fixed momentum direction; ii) focusing in both radial and axial directions, if the configuration field satisfies exactly the eqs. (1.1). The electric

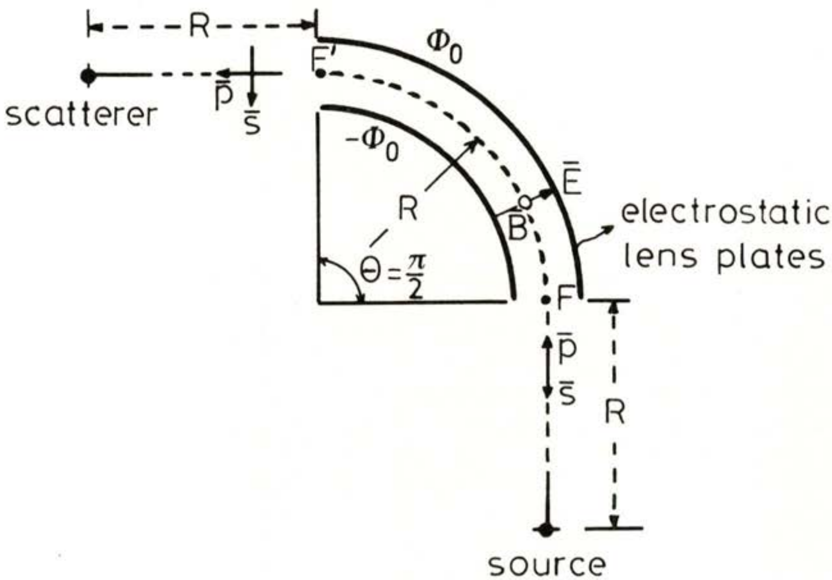

Fig. 1 — Electron trajectories in a $\pi/2$ sector configuration of crossed electric (**E**) and magnetic (**B**) fields.

field can be easily obtained with aid of two concentric spheres of radius R_1 and R_2 ($R_2 > R_1$) at the potentials $-\phi_0$ and ϕ_0 respectively. The magnetic field can be produced by a sector ring magnet with a radially increasing gap between the pole pieces (Fig. 2). The expression $B_z = B_0 (R/r)$ is valid only if the magnetic pole pieces are normal to the spheres defined by the electric lenses [3], [4]. Usually the magnetic field is of the form $B_z = B_0 (R/r)^{1+\alpha}$.

In the following sections these topics are analysed in detail with special emphasis on the influence of α on the characteristics of the polarimeter. It is a numerical analysis and in the calculations we assume that the angular width of the sector field is $\Theta = \pi/2$, the real parts of object and image spaces are located in field free regions and the distance d_1 from the object to the entrance boundary of the sector field is equal to R (Fig. 1).

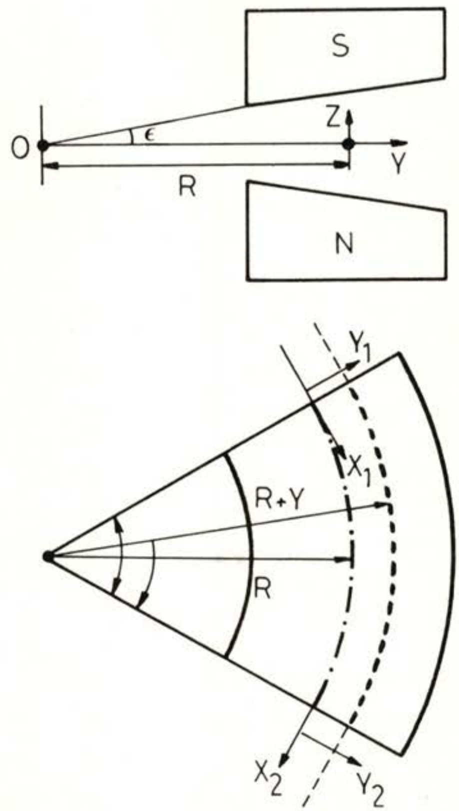

Fig. 2 — Schematic view of the pole pieces of the magnet.

2 — PARTICLES IN PRESENCE OF ELECTROMAGNETIC FIELDS

Consider the motion of an electron of velocity $v = \beta c$ (charge $-e$ and mass $m = m_0 (1-\beta^2)^{-1/2}$) under the influence of a radial electric field E_0 in the plane of its orbit and an axial magnetic field B_0 normal to that plane. Assuming cylindrical symmetry the equilibrium trajectory in the laboratory frame is a circle of radius R given by [3]

$$1/R = 1/\rho_e + 1/\rho_m \ , \qquad (2.1)$$

where

$$1/\rho_e = e\, E_0\, W/(c^2 p^2) \quad , \quad 1/\rho_m = e\, B_0/p \ . \qquad (2.2)$$

In these equations $mc^2 = \gamma\, m_0\, c^2 = W$ is the total relativistic energy and $mv = mc\,\beta = p$ is the momentum.

On the other hand, if the angular width of the sector field is equal to Θ the angle between the spin of the electron and its momentum changes by [3]

$$\Delta\,\sigma = -R\,\Theta/(\rho_e\,\gamma) = -(R/\rho_e)\ (1-\beta^2)^{1/2}\ \Theta \ . \qquad (2.3)$$

Eqs. (2.1) and (2.3) allow the conversion of longitudinal into transverse polarization for all values of the energy.

This transverse polarization P of the electrons is determined by recording the left-right asymmetry A in Mott scattering from spinless nuclei [4]. This asymmetry is $A = S\ P$ where the Sherman function S is known [5]. This function depends on the electron energy and the scattering angle.

The focusing properties of the sector field can be derived by studying the electron motion slightly displaced from the equilibrium orbit of radius R. The coordinates of the electron are s, representing the displacement along the equilibrium orbit, and y and z representing the displacements parallel to the curvature radius and perpendicular to the plane of the equilibrium orbit, respectively. For small displacements the equation of motion are [6]

$$d^2 y/ds^2 = -K_y^2(s)\, y \ , \ d^2 z/ds^2 = -K_z^2(s)\, z \qquad (2.4)$$

where the coefficients $K_y^2(s)$ and $K_z^2(s)$ are

$$\begin{aligned} K_y^2(s) &= R^{-2}\,[\,1 + (R/\rho_e)^2\,(1-\beta^2)\,] - K_z^2(s) \\ K_z^2(s) &= -R^{-2}\{(R/\rho_e)\ [\,1 + (\partial E_r/\partial r)_{r=R}\ (E_0/R)^{-1}\,] \qquad (2.5) \\ &\quad + (R/\rho_m)\ (\partial B_z/\partial r)_{r=R}\ (B_0/R)^{-1}\,\} \ . \end{aligned}$$

To determine the position of the electron source and its image we apply the usual techniques of electron optics. Assuming that the real parts of object and image spaces are located in field free regions and that within the polarimeter the trajectories are determined by the solutions of eqs. (2.4), the condition for stigmatic imaging is found to be [3]

$$1/d_1 + 1/d_2 = [\, K - (K\, d_1\, d_2)^{-1} \,] \, \tan K R \Theta \qquad (2.6)$$

where d_1 is the distance from the object (electron source) to the entrance boundary of the sector field and d_2 the distance from the exit boundary to the image. The condition that the object and image distance (focusing distances) should be equal is also obtained directly from eq. (2.6), i.e.,

$$K\, d_1 = K\, d_2 = \begin{matrix} +\cot \\ -\tan \end{matrix} \, (K R \Theta/2) \qquad (2.7)$$

The positive and negative signs refer to negative and positive lateral magnifications respectively. When object and image lie in their respective real plane, i.e. real images are obtained, the positive sign is taken in eq. (2.7) expressing the fact that the image is inverted. Thus, if the electric field and magnetic field are given by eq. (1.1) and $\Theta = \pi/2$ the previous equations show that $d_1 = d_2 = R$.

3 — RESULTS

For $\alpha \neq 0$ eqs. (2.5) become

$$\begin{aligned} K_y^2(s) &= R^{-2} \, [\, (R/\rho_e)^2 \, (1-\beta^2) - \alpha \, (1 - R/\rho_e) \,] \\ K_z^2(s) &= R^{-2} \, [\, 1 + \alpha \, (1 - R/\rho_e) \,] \end{aligned} \qquad (3.1)$$

and the condition for astigmatic focusing is evidently $K_y = K_z$, i.e.

$$(1-\beta^2) \, (R/\rho_e)^2 + 2\,\alpha \, (R/\rho_e) - (1 + 2\,\alpha) = 0 \, . \qquad (3.2)$$

From these equations we see that for $\alpha \neq 0$ it is not possible to have simultaneously the conversion of electron longitudinal polar-

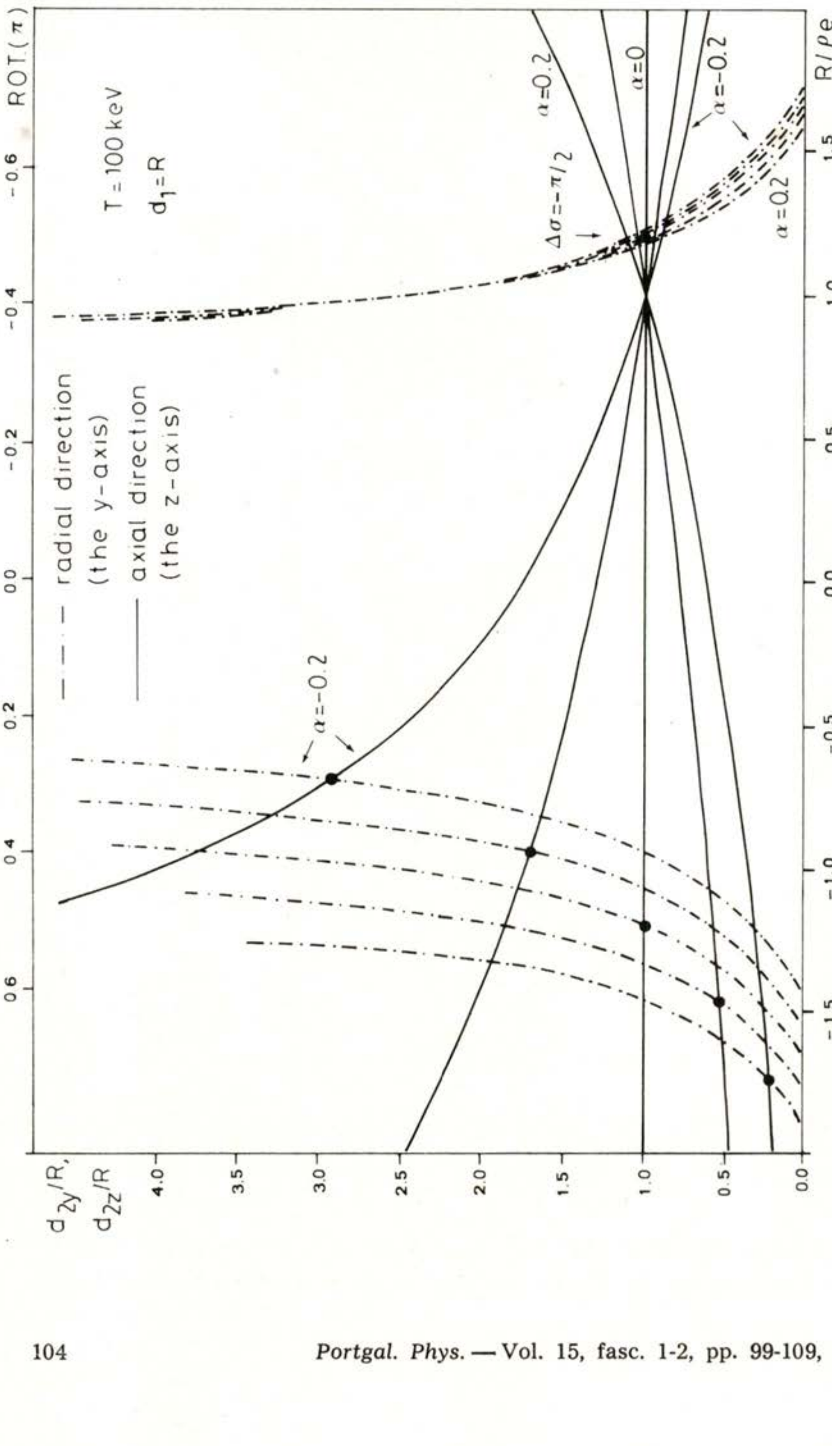

Fig. 3 — Radial and axial focusing ratios d_y/R and d_z/R at 100 keV plotted as a function of R/ρ_e for a range of values of the magnetic field index α. The top scale gives the $\Delta\sigma$ values.

ization into transverse polarization and astigmatic focusing. The condition $d_1 = d_2 = R$ is no longer valid and the distances $d_2\,(d_{2y}, d_{2z})$ can be obtained from eq. (2.6). Writing

$$K\,d_2 = -\,[\,2\tan(K R\,\Theta/2) + K d_1\,(1 - \tan^2(K R\,\Theta/2))\,] \cdot [\,1 - \tan^2(K R\,\Theta/2) - 2 K d_1 \tan(K R\,\Theta/2)\,]^{-1} \tag{3.3}$$

the radial and the axial focusing ratios d_{2y}/R and d_{2z}/R can be determined as a function of R/ρ_e for a range of values of the magnetic field index α. Assuming that $\Theta = \pi/2$ and $d_1 = R$, these ratios were computed for different electron energies. This is shown in Figs. 3, 4 and 5. These Figs. are quite general and so they can be used to predict the anastigmatism of any polarimeter where $\Theta = \pi/2$ and $d_1 = R$. The top scale of these Figs. gives the $\Delta\,\sigma$ values which were calculated from eq. (2.3).

In all Figs. we see that for a given α the intersection of the two curves is an astigmatic focusing point $(d_{2y} = d_{2z})$. To each astigmatic point corresponds a value of R/ρ_e. If the electron energy is small we see in Fig. 3 that the value of $(R/\rho_e) = 1.2$ corresponding to the astigmatic point is roughly equal to the value needed to rotate the spin by $-\pi/2$. However for higher energies (Figs. 4 and 5) this situation is no longer true.

To further illustrate this point, let us consider an electron of 1000 KeV and $\alpha = -0.1$ (Fig. 5). To achieve a spin rotation of $-\pi/2$ we need $(R/\rho_e) = 2.97$ whereas double focusing occurs for $(R/\rho_e) = 3.65$. In other words setting $(R/\rho_e) = 2.97$ we obtain $(d_{2y}/R) = 1.67$ and $(d_{2z}/R) = 0.61$ which means that the image of the source rather than being a point is spread over a 1.06 R region. Obviously, the scatterer must be positioned within this region. Clearly, a situation like that implies two things. For a scatterer big enough to maximize the number of scattered electrons the spread in the electron incoming angles will imply a large error in the estimation of the Sherman function. On the other hand, if the scatterer is small in order to avoid the previous problem then there will be a drastic reduction of the number of scattered electrons. This condition implies a large statistical error

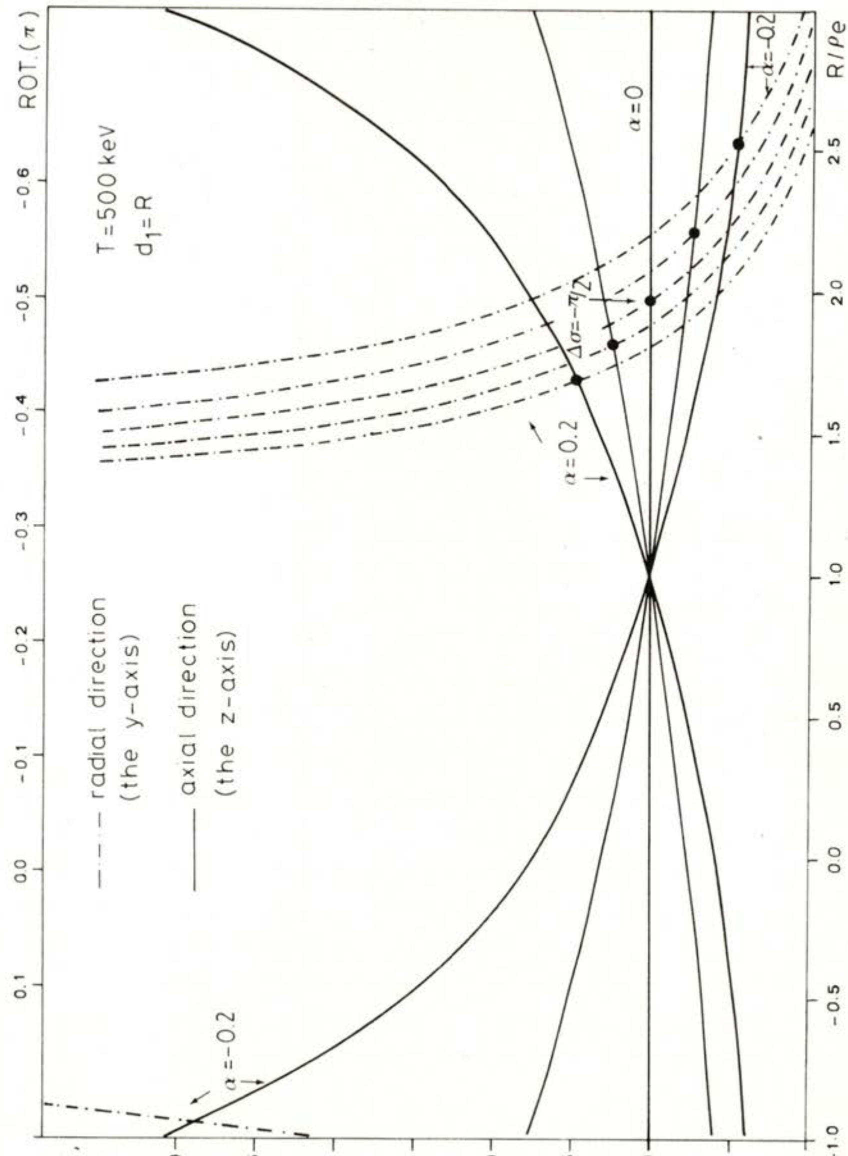

Fig. 4 — Radial and axial focusing ratios d_y/R and d_z/R at 500 keV plotted as a function of R/ρ_e for a range of values of the magnetic field index α. The top scale gives the $\Delta\sigma$ values.

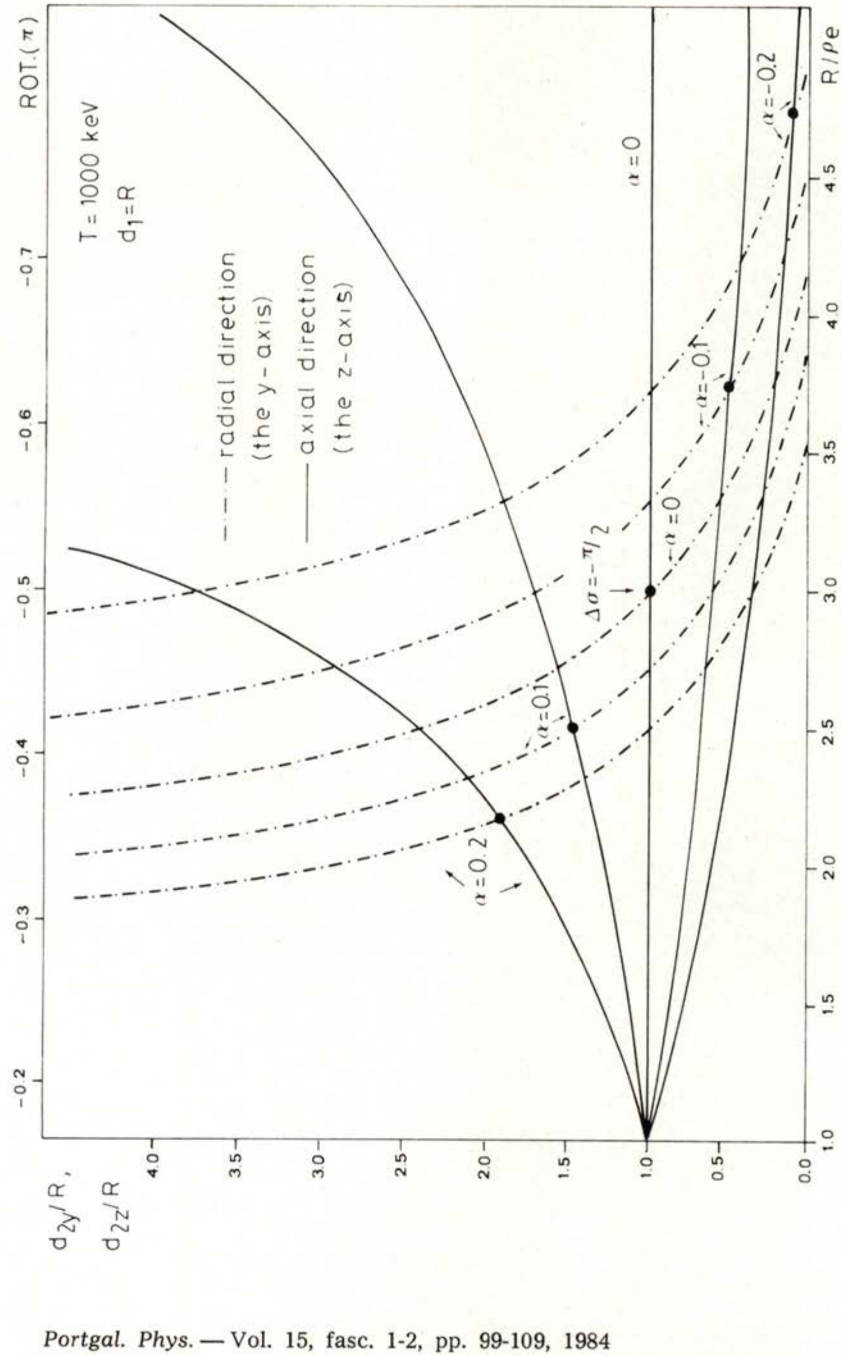

Fig. 5 — Radial and axial focusing ratios d_y/R and d_z/R at 1000 keV plotted as a function of R/ρ_e for a range of values of the magnetic field index α. The top scale gives the $\Delta\sigma$ values.

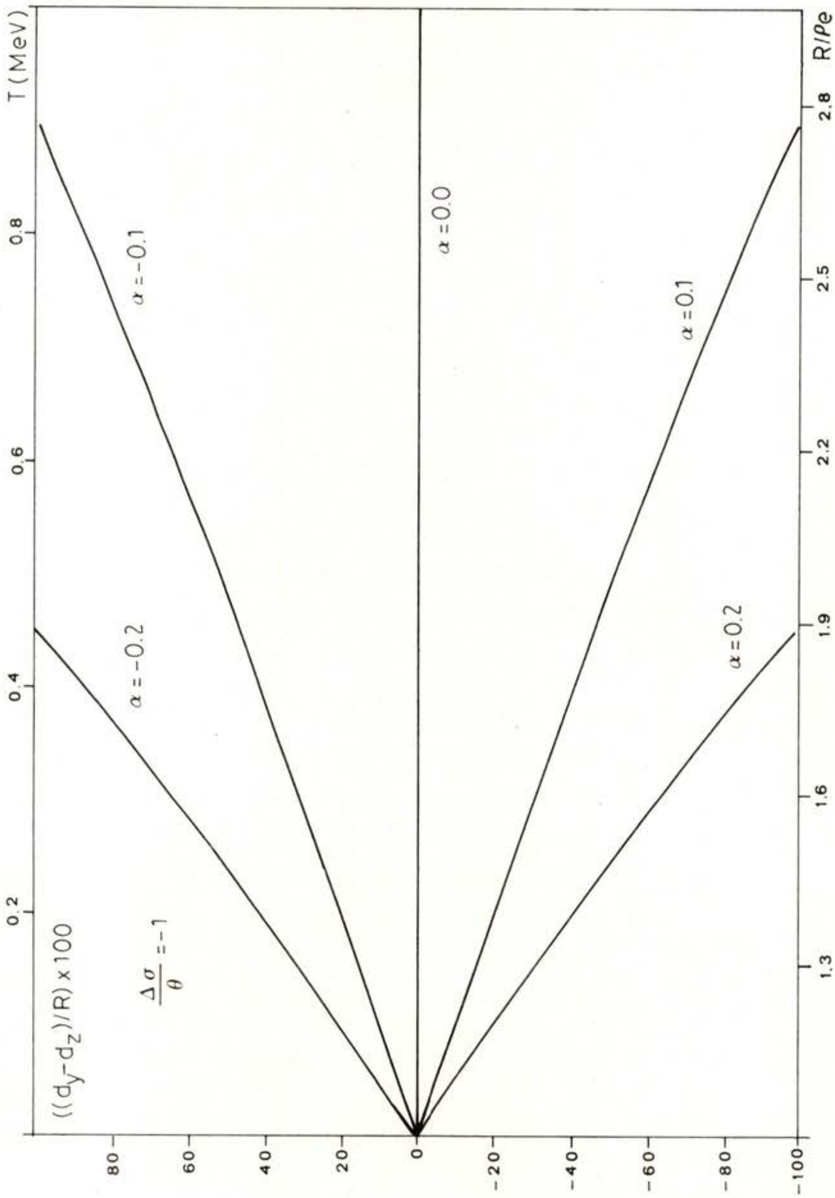

Fig. 6 — The anastigmatism of the lenses at $\Delta\sigma/\Theta = -1$ plotted as a function of the energy and of the index α. The R/ρ_e values are taken from eq. (2.3).

in the measurements. Fig. 6 shows for an electron spin rotation of $-\pi/2$ the anastigmatism of the polarimeter. We see that for a given $\alpha \neq 0$ the anastigmatism is energy dependent.

4 — CONCLUSIONS

We would like to emphasize that our conclusions come from numerical calculations and they can be summarized as follows:

1 — To transform electron longitudinal polarization into transverse polarization by an astigmatic ($d_y = d_z$) polarimeter we must have $\alpha = 0$.

2 — The anastigmatism of the polarimeter depends on the magnetic field index α, and for $\alpha \neq 0$ the focusing caracteristics of the apparatus do not remain constant when we change the electron energy.

3 — For small α and low values of electron energies it is not critical to know accurately the shape and homogeneity of the magnetic field.

4 — For higher energies and $\alpha \neq 0$ the distortions of the image introduce errors in the measurements and they are energy dependent.

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