ERRATA AND ADDENDA: FRUSTRATED SPIN SYSTEMS

(Portgal. Phys. 15, 9-54 (1984))

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(Received 19 October 1984)

The relevant set of symmetry operations on the Potts spins, taking q possible values, is not the whole permutation group of q objects but the q dimensional cyclic group (see paragraph preceding eq. 3.4) which is abelian, and which has the desired property that all elements are traceless except for the identity, in a q-dimensional matrix representation (see paragraph following eq. 3.8).

One should stress that the simplest gauge invariant object that can be constructed from the gauge variables $\psi(\mathbf{r},\mathbf{r}')$ is the plaquette function $\Pi_p \psi$, or the matrix product of the gauge variables taken around a plaquette. The trace of this object (or the quantity ϕ_p, eq. 3.11) is only one of the q - 1 independent scalar quantities that can be constructed from $\Pi_p \psi$. The partition function, however, will depend upon all such gauge invariant quantities.

Define the vector

$$f_p^{\alpha} = (1/q) \sum_{\beta,\gamma} \delta_{\beta,\gamma+\alpha-1} \left(\prod_p \psi\right)_{\beta\gamma} \qquad \alpha,\beta,\gamma = 1,\dots q.$$

Note that the first element is just $1/q$ times the trace itself, and $\Sigma\, f_p^{\alpha} = 1$. In specifying the frustration configuration, one may now specify each of these vectors, as, say, taking the values $\hat{\phi}_p$.

Thus, in eq. (3.14) $\hat{\phi}_p$ should be substituted for the scalar ϕ_p that appears there. Similiarly, eq. (3.17) should read,

$$Z \{ \hat{\phi}_p \} = \Omega_M \sum_{\{\psi\}} \prod_p \delta (\hat{\phi}_p , \mathbf{f}_p) \exp [k \sum_{(\mathbf{r},\mathbf{r}')} \psi_{00} (\mathbf{r} , \mathbf{r}')]$$

The Kroenecker delta $\delta (\hat{\phi}_p , \mathbf{f}_p)$ can be expressed conveniently as

$$\delta (\hat{\phi}_p , \mathbf{f}_p) = \hat{\phi}_p^\dagger \, \mathbf{f}_p$$

and, up to an infinite constant factor, as

$$\lim_{K_p \to \infty} \exp [K_p \, \phi_p^\dagger \, \mathbf{f}_p]$$

giving, in place of eq. (3.19), (3.20),

$$Z \{ \hat{\phi}_p \} = \Omega_M \lim_{K_p \to \infty} \sum_{\{\psi\}} \exp [K \sum_{\mathbf{r},\mathbf{r}'} \psi_{00} (\mathbf{r} \;\; \mathbf{r}') + K_p \sum \hat{\phi}_p^\dagger \, \mathbf{f}_p]$$

The Duality Transformation given in the Appendix is correct for the unfrustrated case (setting all $\phi_p = 1$ in the notation used there). The generalization to the frustrated case, however, does not follow along the same lines as the Ising model. In particular, the statement that it can be accomplished by the replacement $K_p \to - K_p / (q - 1)$ turns out to be incorrect in general. Instead of eq. (4.15) one should have, using the following parameterization of

$$\hat{\phi}_p^\alpha = \delta_{\alpha, r_p +1} \quad , \quad r_p = 0 , \dots q - 1 \quad \text{and} \quad - r_p \equiv q - r_p \; ,$$

that: i) in the case of one frustration at a plaquette p dual to the site $\tilde{i}$

$$(q - 1)^{-1} < q \, \delta_{\sigma_{\tilde{i}} , 1} - 1 >_{k^*}$$
$$= \lim_{K_p \to \infty} \frac{Z \{ r_p \neq 0 , \text{ all } r_q \; (q \neq p) = 0 \}_{k , k_p}}{Z \{ \text{all } r = 0 \}_{k , k_p}}$$

ii) in the case of two frustrations at plaquettes p, q dual to sites $\tilde{i}$, $\tilde{j}$:

$$(q-1)^{-1} < (q\,\delta_{\tilde{i},1}-1)\,(q\,\delta_{\tilde{j},1}-1) >_{K^*} =$$

$$\lim_{K_p \to \infty} [\, Z_{K,K_p} \{ r_p \neq 0,\, r_q = -r_p,\, r_s (s \neq p, q) = 0 \} + (q-2) \cdot$$

$$\cdot Z_{K,K_p} \{ r_p \neq 0,\, r_q \neq -r_p,\, r_s (s \neq p, q) = 0 \}] \,/\, Z_{K,K_p} \{ \text{all } r = 0 \}$$

iii) in the case of three frustrations at p, q, s dual to $\tilde{i}$, $\tilde{j}$, $\tilde{k}$

$$(q-1)^{-1} < \prod_{\mu = \tilde{i}, \tilde{j}, \tilde{k}} (q\,\delta_{\sigma_\mu, 1} - 1) > =$$

$$\lim_{K_p \to \infty} [\,(q-2)\, Z_{K,K_p} \{ r_p + r_q + r_s = 0,\, r_t\, (t \neq p, q, s) = 0 \}$$

$$+ (q-1)\, (Z_{K,K_p} \{ r_p + r_q = 0,\, r_s \neq 0,\, r_t\, (t \neq p, q, s) = 0 \} + \text{permutations})$$

$$+ (q^2 - 6q + 6)\, (Z_{K,K_p} \{ r_p + r_q + r_s \neq 0,\, r_p + r_q \neq 0,\, r_s \neq 0,\, r_t\, (t \neq p, q, s)$$

$$= 0 \} + \text{permutations})] \,/\, Z_{K,K_p} \{ \text{all } r = 0 \}$$

The cases with more than three frustrations are even more complicated. One should be able to invert these equations for the partition functions, but this has not yet been accomplished.

I would like to thank Drs. E. J. S. Lage, L. Banyai and Prof. Wegner for their useful comments.