RESONANT ABSORPTION OF GRAVITATIONAL WAVES IN THE PAST OF THE UNIVERSE

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ABSTRACT — The way the cyclotron absorption of gravitational waves varies with the cosmological red shift is analised.

One concludes that waves in the proto interstellar and interstellar medium with frequency near the double of the electron Larmor frequency $2\,\omega_L$ were never significantly absorbed in the period between the epoch when the reaction $\gamma \rightarrow e + \overline{e}$ ceased to occur spontaneously and the present epoch.

It has recently been shown [1] that in a non-collisional magnetized plasma, there is a weak cyclotron damping of gravitational waves propagating parallel to the magnetic field which permeates the plasma. For a weak collisional regime this effect has been shown to increase with collisions [2].

The existence of an electromagnetic microwave background with temperature of 3 K has been known for many years [3]. This effect is due to the recombination of hydrogen when the temperature of the Universe was 4000 K.

We therefore address ourselves the question: Was there an epoch in the past, when the Universe was not transparent to gravitational radiation in the resonant band near $2\,\omega_L$ (ω_L being the electron Larmor frequency for the interstellar medium and the media which in the past gave rise to it) due to this phenomenon?

If the answer to this question were affirmative this could be a mechanism to contribute for a fossil gravitational wave background.

However, we conclude by this study that the answer is negative. For the sake of simplicity, we assume that from the epoch of galaxy formation until now galaxies did not have a significant evolution as far as the values of density, temperature and magnetic field are concerned. The interstellar medium is therefore assumed to be a fossil which dates from that epoch.

We also assume a frozen-in magnetic field which permeated the pre-galactic medium along its evolution.

We assume that the typical values for the collisional frequency (ω_c), electron Larmor frequency (ω_L), temperature (T) and number density (n) for the interstellar medium at the time of galaxy formation where identical to the present ones, i. e.,

$$\omega_{c_G} \sim 6 \times 10^{-4} \text{ Hz} , \ \omega_{L_G} \sim 20 \text{ Hz} , \ T_G \sim 10^4 \text{ K} , \ n_G \sim 10^4 \text{ cm}^{-3} \ (^*),$$

The previous evolution of ω_c , ω_L and n , until such epoch, were given (using the standard Friedman model) [3] as a function of the temperature T by:

$$\begin{aligned} \omega_c &\simeq 6 \times 10^{-10} \ T^{3/2} \ \text{Hz} \\ \omega_L &\simeq 2 \times 10^{-7} \ T^2 \ \text{Hz} \\ n &\simeq 10^{-8} \ T^3 \ \text{cm}^{-3} \end{aligned} \qquad (1)$$

From this, one can see that in the past $\omega_c << \omega_L$ and therefore, one is justified in using a non-collisional model like the one due to Macedo & Nelson [1]. This model does not take into account the electron-positron pair production (Vlasov's equation does not include terms which are due to it) and therefore, this analysis is restricted by the condition that the temperature of the Universe be lower than $T_{e\text{-}\bar{e}}$ given by

$$k\, T_{e\text{-}\bar{e}} \sim 2\, m_e\, c^2 \qquad (2)$$

where m_e is the electron rest mass. This condition amounts to $T < 10^{10}$ K .

In Macedo & Nelson's model, the frequency of gravitational waves propagating in a magnetized plasma of density n has a real part ω_r and an imaginary part ω_i (which is responsible for the wave damping), which is given by [2]:

$$\omega_i \sim 10^{-5} \ n\, T^{1/2}\, m_e^{3/2}\, \omega_L^{-1} \ \exp\left[- m_e\, \lambda^2 \,(\omega_r - 2\, \omega_L)^2 / k\, T \right] , \qquad (3)$$

(*) The subscript G means – at the time of galaxy formation.

where λ is the wavelength; in the vicinity of $2\,\omega_L$, where the exponential approaches 1, (3) reduces to

$$\omega_i \sim 10^{-5}\; n\, T^{1/2}\; m_e^{3/2}\; \omega_L^{-1} \tag{4}$$

Using (1) in (4) one gets the behaviour of this imaginary frequency with the temperature of the Universe in the period between the ceasing of $e + \bar{e}$ pair production and the period of galaxy formation

$$\omega_i \sim 10^{-47}\; T^{3/2} \tag{5}$$

If we define the damping frequency ω_d as the inverse of the time t_d it takes for the intensity of the waves to reduce to half its value, one has

$$\omega_d \simeq 1.4 \times 10^{-47}\; T^{3/2} \tag{6}$$

We define a typical frequency ω_τ as the inverse of the time a graviton takes to cross a typical scale distance d_τ which at the time of galaxy formation is of the size of a galaxy ($\sim 10^{23}$ cm).

$$\omega_{\tau_G} = c/d_{\tau_G} \tag{7}$$

Using a Friedman standard model, this distance scale in the past of the Universe (before galaxy formation) evolved as

$$d_\tau = d_{\tau_G}\, T_G\; T^{-1} \;, \tag{8}$$

or

$$d_\tau \sim 10^{27}\; T^{-1} \tag{9}$$

This means that

$$\omega_\tau \sim 3 \times 10^{-13}\; T \tag{10}$$

Comparing ω_τ given by (10) with ω_d given by (6), one notices that, for $T < 10^{10}\,K$, $\omega_d << \omega_\tau$.

One therefore concludes that, in the frequency band near the resonant frequency $2\,\omega_L$ there is no significant cyclotron absorption of gravitational waves in the Universe in epochs later than the epoch when e - e pairs ceased to be spontaneously produced (*).

In this period, we therefore conclude that the Universe was transparent in this frequency band and the cyclotron absorption did not contribute to the gravitational radiation background.

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REFERENCES

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(*) Note that for $T = 10^{10}$ K one has $\omega_d \sim 10^{-32}$ Hz and $\omega_\tau \sim 10^{-3}$ Hz. In fact this differs by only one order of magnitude from the value of the Hubble parameter at $T = 10^{10}$ K which was $(\dot{R}/R)_{T = 10^{10}\,K} \sim 10^{-2}$ Hz, supposing that $H_0 \sim 50$ Kms^{-1} Mpsc^{-1} and $q_0 \sim 0.002$ [3].