THE PROBLEM OF SPURIOUS COLOR BOUND STATES IN NAMBU JONA-LASINIO TYPE LAGRANGIANS *

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ABSTRACT - Quark-antiquark states in a chiral invariant four fermion interaction lagrangian without confinement are investigated. We issue a warning regarding the possible existence of color bound states in this type of lagrangians frequently used in low energy hadron physics. Simply ignoring the color terms as a consequence of confinement is not sufficient, since the appearance of bound color states indicates that the physics of confinement has also an effect at low energies.

1. INTRODUCTION

The original Nambu Jona-Lasinio (NJL) model [1] has been mainly designed as a mechanism to create Goldstone bosons via dynamical chiral symmetry breaking in a four fermion interaction. Many authors have used NJL-type lagrangians to describe other mesonic states as well, see e.g. [2-5], although no confinement is included. One usually argues that as long as the two quarks are bound, that is for energies below the mass of two quarks, confinement should not significantly change the results. Also, since the lagrangian does not contain confinement, one simply and arbitrarily disregards the color sector which, if not already existent in the direct terms, appears after fierzing. But how reliable is a model whose langrangian gives rise to color terms in which, for instance, a bound color state appears? We should not expect that confinement only suppresses this state without changing as well the physical meson masses. Just disregarding color terms is certainly not the way to mimic confinement. To see whether the color sector can have the alarming behavior of allowing for bound color mesons or significant strength in the quark-antiquark ($q\bar{q}$) continuum, we

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study, as an example, a simple quartic NJL-type model corresponding to thequark sector of the 't Hooft lagrangian [6, 7], which has been used by several authors [2, 4, 8].

It is not our intention to calculate a complete meson spectrum from an NJL-lagrangian, nor to elaborate on more complicated non-local interactions, but to concentrate upon the significance of the color sector in a specific case which serves as an example. We anticipate our findings by saying that there are indeed color bound states and strength in the $q\bar{q}$ -continuum in this special model. There is no reason to believe that other types of lagrangians without confinement will be free of this type of problems.

In section 2 we present the general properties of the model and calculate the bound and continuum states of the color singlet in sect. 3 and of the color octet in sect. 4. Sect. 5 summarizes the conclusions, and calculational details are discussed in the appendix.

2- GENERAL PROPERTIES OF THE MODEL

The interaction between two quarks in this model is [6, 7]

$$v_{12}=g_{eff}[I+\gamma_5(1)\gamma_5(2)][I-\tau(1)\tau(2)][a+b\Lambda(1)\Lambda(2)] \quad (1)$$

with τ the isospin vector, Λ the Gell-Mann SU(3) color vector and I unit matrices. The coefficients are a=2 and b=-3/8. With this interaction, the Fierz transformed lagrangian becomes (suppressing indices 1 and 2)

$$\begin{aligned}\mathcal{L}(x)=&\bar{\Psi}(x)(i\partial-m_c)\Psi(x) + 2g_{eff}\left[[\bar{\Psi}(x)\Psi(x)]^2 + [\bar{\Psi}(x)i\gamma_5\tau\Psi(x)]^2 + [\bar{\Psi}(x)i\tau\Psi(x)]^2 + [\bar{\Psi}(x)\gamma_5\Psi(x)]^2\right] +\\ &\frac{3}{16}g_{eff}\left[[\bar{\Psi}(x)\Lambda\Psi(x)]^2 + [\bar{\Psi}(x)i\gamma_5\tau\Lambda\Psi(x)]^2 + [\bar{\Psi}(x)i\tau\Lambda\Psi(x)]^2 + [\bar{\Psi}(x)\gamma_5\Lambda\Psi(x)]^2\right] +\\ &\frac{9}{32}g_{eff}\left[[\bar{\Psi}(x)i\tau\sigma_{\mu\nu}\Psi(x)]^2 + [\bar{\Psi}(x)\sigma_{\mu\nu}\Lambda\Psi(x)]^2\right] \end{aligned} \quad (2)$$

where Ψ and $\bar{\Psi}$ are the quark and anti-quark fields, and the effective coupling g_{eff} is constant. Although the point interaction is an approximation to the lagrangian derived by 't Hooft, we think that the structure of the forces, which we keep, is more important than their range. We take the current quark mass m_c=0. Calculations are done to one-loop order , i.e. to Hartree-Fock approximation. The lagrangian contains explicitly color-octet terms.

As in the case of the NJL model [1], the quartic interaction lagrangian is non-renormalizable and we take a covariant cutoff L for the loop momenta in one-loop approximation. As usual, the quark self-energy leads in Hartree-Fock approximation to the self-consistency equation

$$1= 12\, g_o \int_o^{\lambda^2} \frac{dy\, y}{4\pi^2}\frac{1}{y+1} = \frac{12}{4\pi^2}\, g_o\, [\, \lambda^2 - \ln(1+\lambda^2)\,] \qquad (3)$$

with the dimensionless quantities $g_o = g_{eff}.m^2$ and $\lambda^2 = L/m^2$ where the factor 12 arises from taking traces over spin, isospin and color matrices and from the coupling. In addition to the self consistency equation, the cutoff and the quark mass are related through the expression for the weak decay constant $f_\pi = 93$ MeV of the pion, calculated the same way as in the NJL model [9-11]

$$f_\pi^2 = \frac{3m^2}{4\pi^2}\,[\, \ln(1+\lambda^2) - \frac{\lambda^2}{1+\lambda^2}] \qquad (4)$$

3- COLOR SINGLET STATES

We proceed to determine the poles of the propagators corresponding to the different couplings from the Bethe-Salpeter equation to one-loop order. The meson propagators have the structure

$$G_i = [\, 1 - J_i\,(\rho^2)\,]^{-1} \qquad (5)$$

(omitting couplings and with $\rho^2 = k^2/4m^2$, k^2 = square of invariant meson mass), where $J_i\,(\rho^2)$ is the fundamental $q\bar{q}$-loop for the i-th type of meson and 1 stands for the self consistency eq. (3). In analogy to the NJL model, the pion emerges as a Goldstone mode in the course of dynamical breaking of chiral symmetry, and the scalar-isoscalar meson appears as a $q\bar{q}$ bound state with mass 2m, at the threshold of the $q\bar{q}$-continuum, with the $q\bar{q}$-loops J_π and J_σ given in the appendix.

The residue at the pole of G_π is the square of the strength (henceforth always denoted as strength) of the π-$q\bar{q}$ coupling and is given by m^2/f_π^2. In the case of the scalar-isoscalar mode, the σ-$q\bar{q}$ strength is calculated in [12] and reduces to m^2/f_π^2 for large λ^2 . In the $q\bar{q}$-continuum $\rho^2>1$ the strengths S_i of the pseudoscalar-isovector and scalar-isoscalar modes are related to the cut of J_i along the imaginary axis.

The pseudo scalar-isoscalar mode is not a bound state (see appendix). A numerical test shows that the scalar-isovector mode has no pole for cutoffs $\lambda^2 > 0.1$. As in the NJL-model, only two bound states appear, the pion and the scalar-isoscalar, for any value of the cutoff. For $\lambda^2 \doteq 3.7$ (corresponding to m = 386 MeV), a value taken in previous works [11,12], all other mesons have their strength distributed in the $q\bar{q}$-continuum [12]. The corresponding strengths are given in the appendix.

4-COLOR OCTET STATES

We turn now to the discussion of the color octet. The $\sigma_{\mu\nu}$ terms lead to vector modes in the Bethe-Salpeter equation to one-loop order. Similarly to [11], the vector-isoscalar mode has never a pole, also in the color sector. However, the vector-isovector mode shows poles whenever the cutoff $\lambda^2 < 1.5$, corresponding to quark masses m > 600 MeV. Another pole appears for the pseudoscalar-isovector octet for $\lambda^2 < 0.2$ or m>2700 MeV. The other octet modes have no poles. We observe that the disturbing poles appear here at constituent quark masses larger than the commonly used value m ≈ 300...400 MeV.

We find that there is some strength in the $q\bar{q}$-continuum (see expression in the appendix). We checked numerically that the strength of the scalar octet states is smaller than that of the corresponding singlet sates (see table 1). We note that the 't Hooft lagrangian favors the color singlet states to a certain degree, except for the vector-isovector mode, which does not exist in the color singlet. To suppress the color octet states completely, one would of course need to include confinement.

Mode	Singlet	Octet	Ratio Singlet/Octet
Scalar-isoscalar	9.97	1.01	9.91
Pseudoscalar-isovector	10.20	1.34	7.61
Scalar-isovector	0.88	0.19	4.63
Pseudoscalar-isoscalar	0.67	0.22	3.12
Vector-isovector	0.	finite	∞

The integrated strengths in the singlet and octet modes (including residues of poles in the singlet) and their ratios, calculated using the cutoff $\lambda^2 = 3.7$.

5- CONCLUSIONS

Our intention was to exhibit the shortcomings of NJL-type models without confinement used in low-energy hadron physics. We do not adhere to the common opinion that simply ignoring the color terms in the lagrangian is sufficient as a consequence of confinement in the low-energy range, since the appearance of color bound states indicates that confinement should also act down to energies below twice the quark mass, the threshold of the $q\bar{q}$-continuum. Therefore, one should expect that physical (uncolored) bound states are also affected by confinement. In the special example considered, color bound states appear only for constituent quark masses larger than the frequently used values of m = 300...400 MeV, and the strengths of color mesons are somehow suppressed as compared to the color singlet mesons with the same quantum numbers. However, there is no guarantee that in other NJL-lagrangians color bound states could not appear for the usual quark masses. Also, the quark mass values of m ≈ 600 MeV at which a bound color vector mode starts to appear in this study is dangerously close to the usually accepted value. The suppression of color meson strengths is not very strong and could also get worse in other models. This, however, might be acceptable since confinement really does act in the $q\bar{q}$-continuum.

6- APPENDIX

In the color singlet, the $q\bar{q}$-loop expressions for the pseudo scalar-isovector (π) and scalar-isoscalar (σ) modes are:

$$J_\pi(\rho^2) = \frac{12\, g_\rho}{4\pi^2} \int_0^{\lambda^2} dy\, y \int_0^1 dx\, \frac{[y + \rho^2(1-x^2)+1]}{[y + \rho^2(x^2-1)+1]^2} \qquad (A1)$$

$$J_\sigma(\rho^2) = \frac{12\, g_\rho}{4\pi^2} \int_0^{\lambda^2} dy\, y \int_0^1 dx\, \frac{[y + \rho^2(1-x^2)-1]}{[y + \rho^2(x^2-1)+1]^2} \qquad (A2)$$

The integrals in (A1) and (A2) are explicitly evaluated in ref. [12]. The corresponding strengths in the $q\bar{q}$-continuum are given by

$$S_i(\rho^2) = 2g_{eff}\, \mathrm{Im}[(1-J_i(\rho^2))^{-1}], \quad i = \pi,\sigma \qquad (A3)$$

The pseudoscalar-isoscalar mode is not a bound state, since the corresponding $q\bar{q}$-loop has the opposite sign of $J_\pi(\rho^2)$ and is therefore always negative, implying that there is no pole in the propagator, see eq. (5). The respective strength in the continuum has the form

$$S_\eta(\rho^2) = 2g_{eff}\, \mathrm{Im}[(1-J_\pi(\rho^2))^{-1}] \qquad (A4)$$

The scalar-isovector mode has a $q\bar{q}$-loop equal to the negative of the scalar-isoscalar loop $J_\sigma(\rho^2)$. Its strength in the continuum is calculated as

$$S_\delta(\rho^2) = 2g_{eff}\, \mathrm{Im}[(1+J_\sigma(\rho^2))^{-1}] \qquad (A5)$$

In the color octet, the pseudoscalar-isoscalar octet mode is not a bound state, since the corresponding expression for the $q\bar{q}$-loop $J_{\eta\Lambda}$ is proportional to the negative of J_π. The $q\bar{q}$-loop of the scalar-isoscalar mode is $J_{\eta\Lambda} = J_6/16$; J_6 in turn has its pole at $\rho^2=1$ the maximum allowed value. Hence to find a pole, ρ^2 would have to be grater than one, since J_σ is monotonically increasing with ρ^2. We determined numerically the non-existence of bound states for the scalar-isovector mode of the color octet with $q\bar{q}$-loop $J_{\delta\Lambda}$. The pseudoscalar-isovector octet with $q\bar{q}$-loop $J_{\pi\Lambda}$ has also no pole for $\lambda^2 > 0.2$, which corresponds to quark masses smaller than 2700 MeV, according to eq. (4).

The expressions for the strengths in the color octet related to the scalar-isoscalar ($\sigma\Lambda$), pseudoscalar-isovector($\pi\Lambda$), scalar-isovector($\delta\Lambda$) and pseudoscalar-isoscalar ($\eta\Lambda$) modes are given by

$$S_{\sigma\Lambda}(\rho^2) = \frac{3}{16} g_{\text{eff}} \, \text{Im}[(1 - \frac{1}{16} J_\sigma(\rho^2))^{-1}]$$

$$S_{\pi\Lambda}(\rho^2) = \frac{3}{16} g_{\text{eff}} \, \text{Im}[(1 - \frac{1}{16} J_\pi(\rho^2))^{-1}]$$

$$S_{\delta\Lambda}(\rho^2) = \frac{3}{16} g_{\text{eff}} \, \text{Im}[(1 + \frac{1}{16} J_\sigma(\rho^2))^{-1}]$$

$$S_{\eta\Lambda}(\rho^2) = \frac{3}{16} g_{\text{eff}} \, \text{Im}[(1 + \frac{1}{16} J_\pi(\rho^2))^{-1}] \qquad \text{(A6)}$$

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