MODELLING WITH THE COMPUTER AT ALL AGES

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ABSTRACT -The paper discusses how modelling with the computer can help Physics Education, for students in a wide age range, from Primary School to University. Different kinds of computational modelling, including iterative dynamic models, qualitative models and object-oriented models are discussed, with examples. It is argued that the different types of models fit naturally into a developmental sequence, matching modelling at various ages to student's intellectual abilities. A radical re-sequencing of teaching about Mathematics in Physics is proposed. Similar ideas are discussed in Ogborn 1990 and 1991.

1 Making models on the computer

An example is often the best way to see something general. So let us begin by making a model. Consider something which interests most people: how to get money, specifically money for one's department. All departments argue for more money in the coming year than they had in the previous year. Suppose to begin with that the increase is constant in each year. To model this in the modelling system we call 'Cell Modelling System' or CMS (Ogborn and Holland 1986), we define three computational cells which are rather like the cells of a spreadsheet, as in Figure 1.

Each cell has a name of a variable in the first slot, and says how to calculate that variable in the second slot, using other variables if required. Thus 'money' is calculated by adding 'increase' to the current value of 'money'. The cell 'increase' defines a constant value (100).

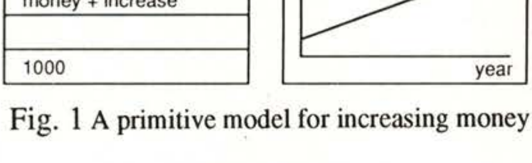

Fig. 1 A primitive model for increasing money

The value of the variable appears in the last slot, putting any initial value required in that slot before running the model. (The third slot, unused in Figure 1, is for comments.) When the model runs, the cell calculations iterate. Thus 'year' increases by unity at each iteration, and 'money' increases linearly from the initial value 1000, adding 100 on each iteration. Any cell can be converted into a graphics display, plotting any one variable against any other. The graph cell in figure 1 shows the linear increase of money with time.

Of course, no department is satisfied with this! The large departments say that the increase should be in proportion to the money they have already. How big the multiplying factor is depends on how fiercely they argue. Figure 2 shows the disastrous model that results.

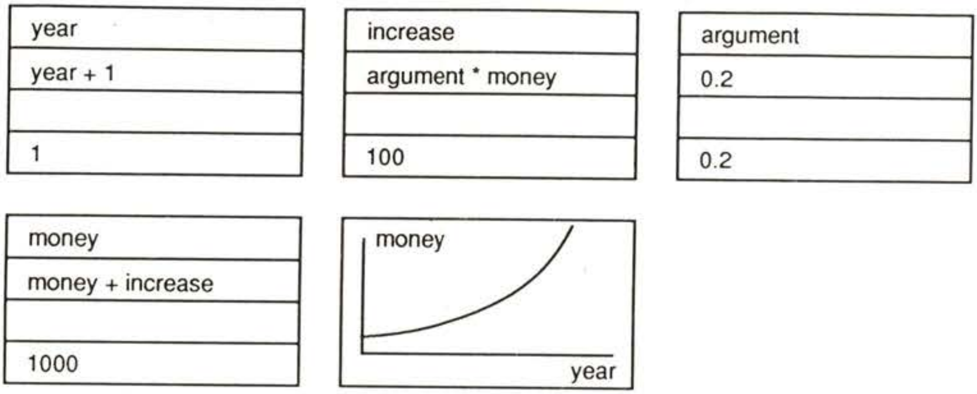

Fig. 2 Exponential increase

If the argument is strong enough to get a 20 percent increase each year, the money grows exponentially. A wise administration realises that this will grow without limit until all the resources of the institution are absorbed. However, exponential growth models are very relevant in for example bacterial growth and epidemics, while the corresponding exponential decay models are relevant in Physics to radioactive decay and to charge on capacitors.

Suppose that a money limit is imposed, such that the increase calculated in Figure 2 is further multiplied by a factor *f* such as (1 - money/limit) so that as the money approaches the limit, the increase is reduced until at the limit it is zero. Figure 3 shows the resulting model.

Figure 3 is logistic growth, common in population studies where a population initially grows exponentially until it starts to run out of food or space. As is now well known, logistic growth models can exhibit chaotic behaviour at large growth rates. If the parameter 'argue' in Figure 3 is made equal to about 2.0, the graph bifurcates and oscillates above and below the 'limit'. At 'argue' = 2.5 the bifurcation has bifurcated, and at about 2.9 the graph goes up and down chaotically (see Figure 4). This behaviour, studied by Fiegenbaum on a pocket calculator, was an important source of our current ideas about chaos. Thus in a few simple steps we have gone from the trivial case of a linear increase, known to any child in secondary school, to near the edge of part of modern mathematics.

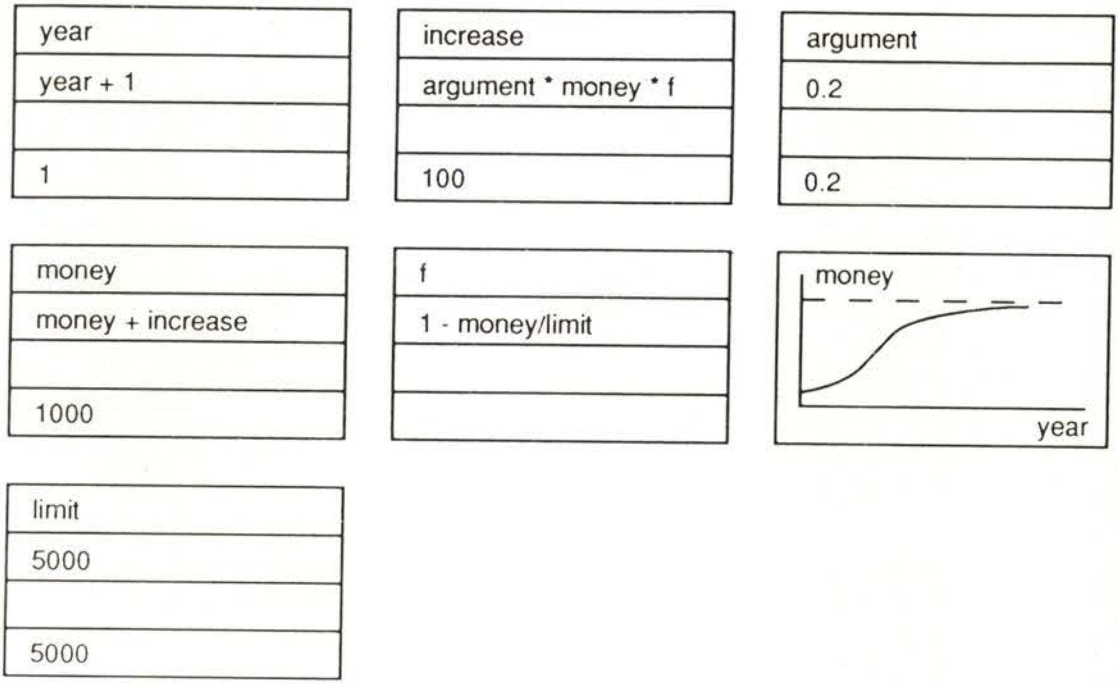

Fig. 3 Logistic growth model.

To take another example, it is not hard to make the Lorenz model of convection currents in a fluid heated from above (see e.g. Marx 1987), as shown in Figure 5.

Air near the ground is warmed and rises, while air high in the atmosphere is cooled and falls. When the warm air has risen it is cooled, and when the cool air has fallen near the ground it is warmed, so the convection can continue.

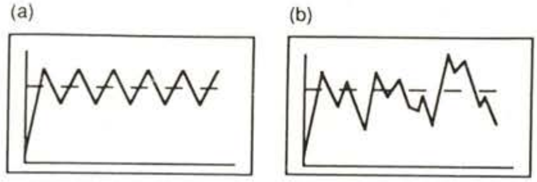

Fig. 4 (a) bifurcation (b) chaos

But if the convection is rapid, warm air is carried over the top of the convection cell without having time to cool, and cool air is carried over the ground without warming. If cool air starts going up and warm air starts coming down, the convection rate will reduce or may even reverse.

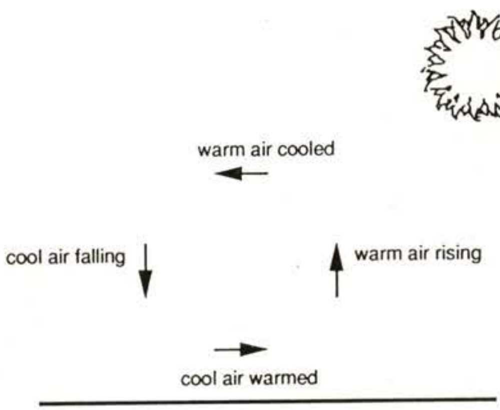

Fig. 5 Convection current in atmosphere

An idealised form of the Lorenz equations relating the rate of circulation to the horizontal and vertical temperature gradients is:

$$dx/dt = 10(y - x)$$
$$dy/dt = -xz + 28x - y$$
$$dz/dt = xy - (8/3)z$$

and is easy to put into a modelling system such as that described here, and will generate the well known Lorenz strange attractor if x, y and z are plotted against one another.

2 DINAMIX: another modelling program

DINAMIX is a modelling system developed in Portugal in the project MINERVA by Vitor Duarte Teodoro of the Technological University in Lisbon. Unlike CMS, in DINAMIX models are expressed directly as differential equations.

As in Figure 6, a model in DINAMIX is written by giving one or more differential equations, specifying initial values and constants, and asking for graphs. A stroboscopic graph option makes it possible to show the motion as well as graphs of speed and position against time.

Figures 7 and 8 show how the very elementary model of Figure 6 can be developed into a model of a harmonic oscillator.

Such a progression suggests how a computer modelling program can be used to teach calculus. Simple models like Figure 6 introduce the idea of a derivative, and relate the derivative to the slope of a graph. In the model of Figure 7 the derivative itself has a derivative, so that the slope of the graph of x against time is continually changing. In Figure 8, the rate of change of velocity is itself determined by x, which is changing. We have a negative feedback loop, from displacement to rate of change of velocity, which determines

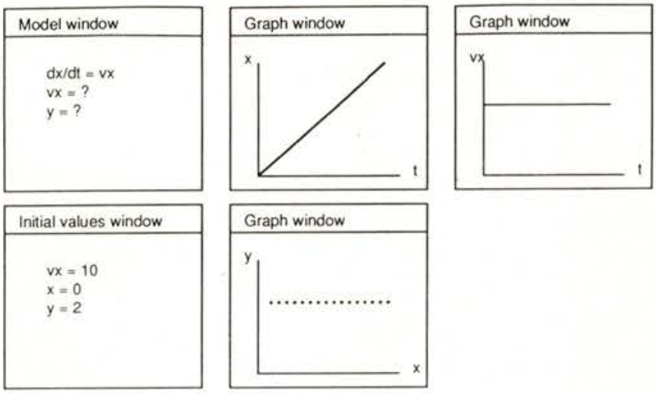

Fig. 6 DINAMIX model of constant velocity

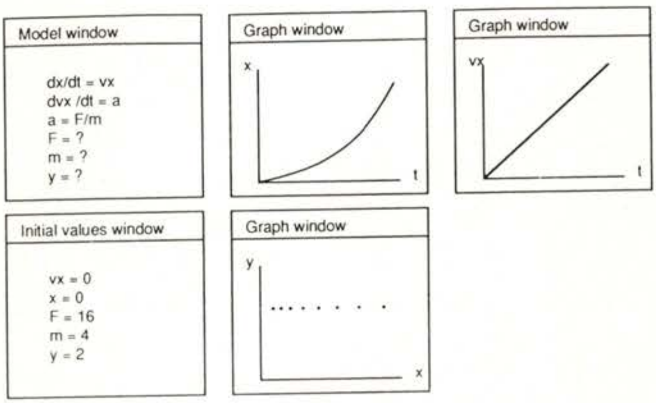

Fig. 7 DINAMIX model of constant acceleration under a constant force

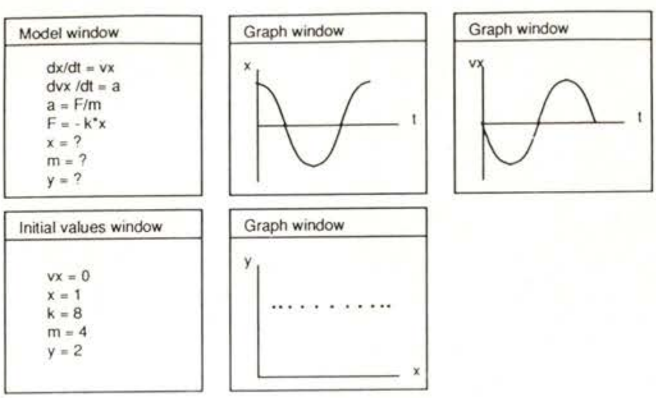

Fig. 8 DINAMIX model of harmonic oscillator

the velocity, which itself determines the displacement. As in other cases we will see later, this is an example of a general system principle:

negative feedback plus delay gives oscillation

3 Iterative dynamic models

All the models we have looked at so far are iterative dynamic models (Roberts et al 1983). Several systems for iterative dynamic modelling exist, amongst them:

System	*machine*	*country*
Dynamic Modelling System	BBC/IBM	UK
Cell Modelling System	BBC/IBM	UK
DINAMIX	IBM	Portugal
STELLA	Macintosh	USA

The Cell Modelling System (Ogborn and Holland 1986) came after our earlier Dynamic Modelling System (Ogborn 1984). STELLA (1987) exploits the graphic capabilities of the Macintosh microcomputer. However, if one has no access to such a system and wants to avoid direct programming, the best solution is to use a commercial spreadsheet (Folha de Calcul) such as EXCEL (Ogborn 1986). Just to make the point, Figure 9 shows a model of radioactive decay built with a spreadsheet.

0.2	decay constant

1000	initial value
800	
640	each value is
512	the previous value
410	minus
328	the decay constant
262	times
210	the previous value
168	
134	
107	

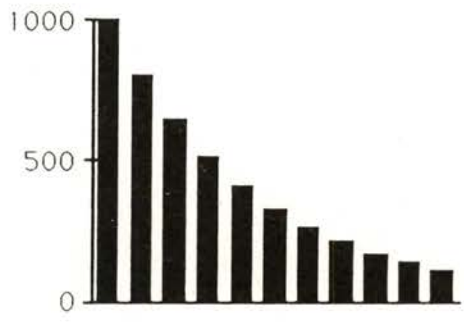

Fig. 9 Spreadsheet model of exponential decay

These modelling systems are suitable for any problem involving solving differential equations or finite difference equations, which is to say, a great number of problems in Science in general and in Physics in particular. Relevant applications include:

Physics
- Mechanics
  - Projectiles, Planetary motion, Oscillator, Relativistic motion
- Electricity and magnetism
  - RC circuits, LR circuits, LRC circuits, particles moving in electric and magnetic fields
- Optics
  - Two slit interference, Diffraction at a slit, Diffraction grating
- Heat
  - Conduction

Chemistry
- Reaction rates
  - Temperature dependence, Concentration dependence
- Equilibria
  - Pressure dependence, Temperature dependence
- Analysis
  - Titration, pH
- Transport
  - diffusion, effusion, pumping

Biology
- Populations
  - exponential growth, limited (logistic) growth
- Ecology
  - competition between species, interdependence of species
- Animal and plant biology
  - energy balance of organisms, animal and plant growth
- Cell biology
  - enzyme reactions, cell growth, nerve impulses

Applied and general problems
- Road traffic
- Home heating
- Electricity supply and demand
- Nuclear power stations
- Diet and slimming

The general concept of iterative dynamic modelling is to identify important variables which describe a system, and formulate how they change in time as a result of the values of other variables and constants. The rules for evolution of a system are thus the rules for computing the next value of each variable. Such equations may have, but will not always (or even often) have, analytic solutions. The tradition in Science teaching has for a long time been to focus exclusively on those equations which do have analytic solutions. Let us compare the advantages and disadvantages of the computational and analytic approaches:

Computational solutions	*Analytic solutions*
Steps close to physical reality	Formal methods of integration
Accessible early in learning	Needs previous mathematics
Adding complexity is easy	Adding complexity is difficult
Only particular solutions	General, manipulable solutions

In general, each step of a computational solution corresponds to some real physical relationship or process, and so has a direct interpretation in reality. The computational process reflects the physical process The same can not be said of the procedures used to obtain analytic solutions: nothing physical corresponds, for example, to the process of integration by parts. For these reasons, computational solutions are accessible earlier in learning, since learning the Physics is also learning the steps in the solution, while to obtain analytic solutions one normally needs other prior mathematical knowledge of functions and of methods of integration.

Because the existence of analytic solutions is very sensitive to the detailed structure of the differential equations (in particular often requiring them to be linear) adding a small real life complexity to a problem may produce a very sharp rise in the mathematical difficulty of solving it. Adding damping to an oscillator makes solving the equations harder, and adding non-linear damping may make an analytic solution impossible. By contrast, in computational solutions, adding complexity will often only add a line or two to a program. Figure 10 fancifully sketches a relation between the difficulty of getting a solution and the amount of reality the model includes.

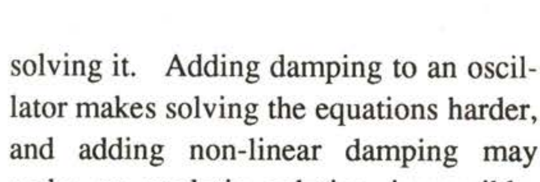

Fig. 10 Difficulty and reality

There is, however, a very good reason for the dominance of analytic solutions. An analytic solution, expressed in a closed form mathematical expression, is quite general and can itself be manipulated and operated on. By contrast, computational solutions are always particular cases. One can move some way towards generality by varying parameters to generate families of solutions, but the computational solutions remain as displays of results rather than manipulable mathematical expressions. Thus analytic solutions will always have an important role to play. They are like diamonds, uniquely valuable, but rare and costly. The question is not whether to do without them in favour of computation, but when and how to include them. Figure 11 contrasts the common traditional sequence of teaching with one which might serve us better (Ogborn 1989).

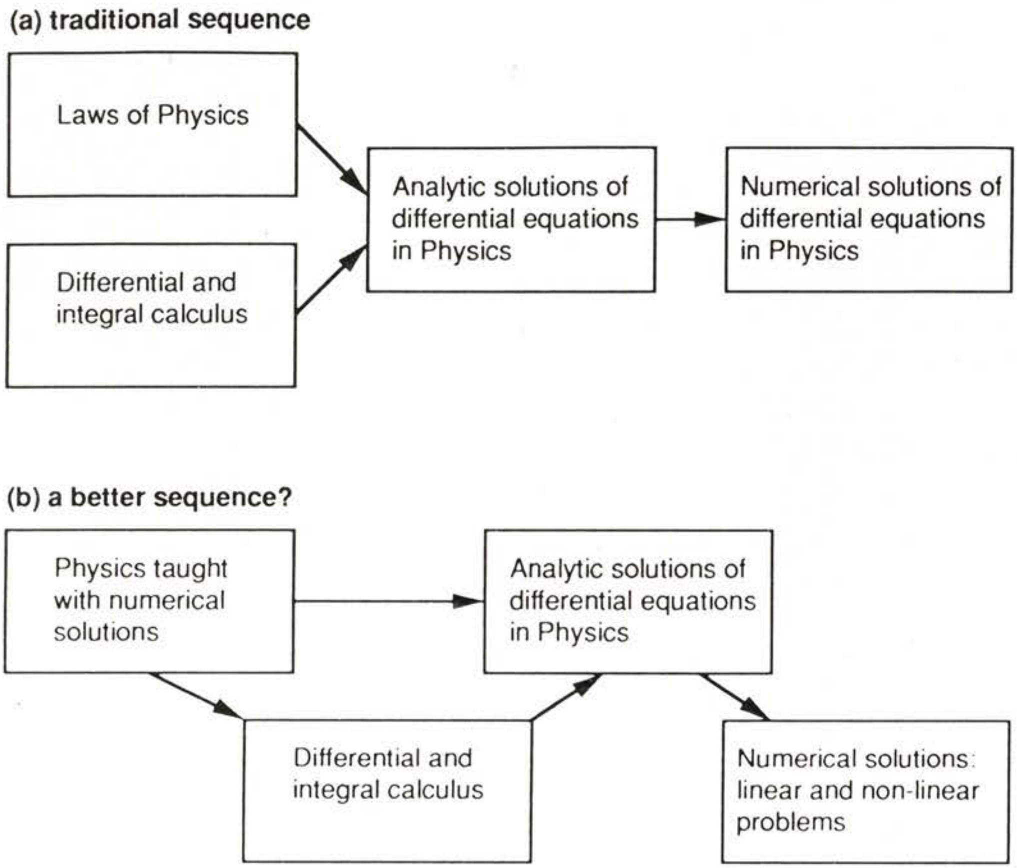

Fig. 11 Traditional and computational learning sequences

Traditionally, we teach Physics and some calculus alongside one another, so as later to be able to develop analytic solutions for differential equations in Physics. Much later, perhaps only in graduate school, is the student introduced to numerical methods. The alternative, which I believe would be better, is to teach Physics by means of some very elementary numerical methods, and to use this to develop the ideas of the calculus so as later to develop analytic methods and numerical methods in parallel.

4 Modelling without mathematics

Up to now, what has been suggested is hardly revolutionary, and fits well with the nature of modern Physics. The next suggestion is however more shocking: it is that we need to begin modelling without, or with the absolute minimum of, mathematics. Consider what is needed if one is to make models of the kind discussed so far:

1 Imagining the world constituted of *variables*
2 Conceiving physical relations as *mathematical relations* between variables
3 Giving appropriate *values* to variables
4 Seeing a model as a *structure* with *possibilities.*

Of these, the first is perhaps the hardest. As Physicists we have become so used to imagining the world as analysable as the interaction of quantitative variables that we forget what a huge step in imagination this is. There is good evidence, supported by commonsense observation, that young students see the world as built of *objects and events,* not as built of variables.

We have built, and tested with students in the age range 12-14 years, a modelling programme which focuses just on imagining variables and the connections between them, without having to specify the form of mathematical relations. It was developed in the project *Tools for Exploratory Learning,* in association with Joan Bliss, Rob Miller, Jonathan Briggs, Derek Brough, John Turner, Harvey Mellar, Dick Boohan, Tim Brosnan, Babis Sakonidis, Caroline Nash and Cathy Rodgers. The background to this project is given in Bliss and Ogborn (1988, 1989). The design of the modelling programme is in Miller et al (1990) and results are discussed in Bliss, Ogborn at al (1992) and Bliss and Ogborn (1992).

The modelling system is called IQON (Interacting Quantities Omitting Numbers). In IQON one creates and names variables, and links them together graphically. Again, the best introduction may be by example.

Figure 12 shows what the previous example of an oscillator looks like when expressed in IQON.

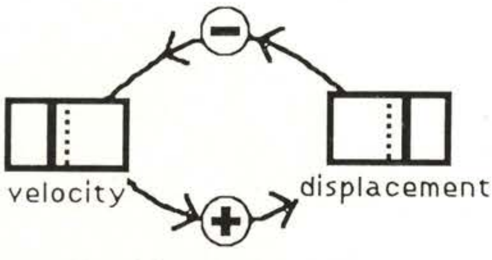

Fig. 12 An oscillator in IQON

A positive velocity progressively increases the displacement, through the 'plus' link. But a positive displacement progressively *decreases* the velocity, through the action of a spring, represented via the 'minus' link. The outcome is that the system oscillates, a example of the principle mentioned before, that negative feedback plus delay gives oscillation. What is shown in Figure 12 is all that the user has to do: to create and name two variables and to link them as shown. No equations are written at all.

However, IQON is also intended for thinking about systems where we have much vaguer ideas about quantities and their relationships. Consider the quality of this very meeting. We may imagine that much depends on the quality of the workshops. If that is high, the participants become happier and happier as the week goes by. But if they are happy they may perhaps participate more actively in workshops, so that the quality of workshops itself increases. Figure 13 shows this idea expressed in IQON.

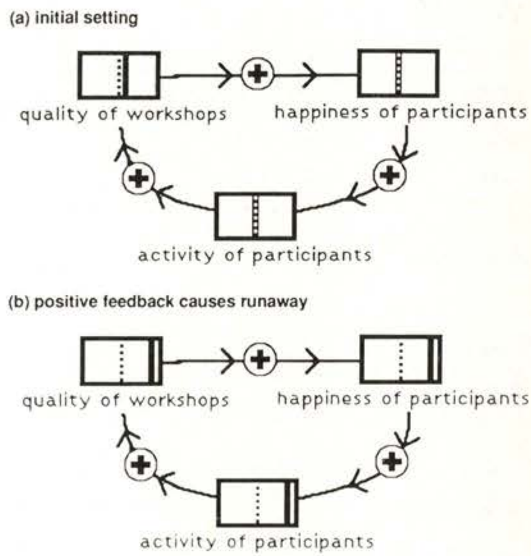

Fig. 13 An IQON model for success of workshops

This model is notably optimistic. It contains positive feedback, so that if as in Figure 13(a) the quality of workshops is somehow increased by a small amount, then after some time all the variables are driven to their positive limits. It does not matter whether the model is correct; what matters is that such effects are possible and will certainly arise in some cases, whatever the details of the system. An increase in global temperature causing melting of polar ice, which by reducing reflectivity increases the energy absorbed from the

Sun and so leads to a further increase of global temperature is an example.
In its present implementation, all IQON variables are alike. Any input from other variables simply modifies the rate of increase or decrease of a variable. Each has a central 'neutral' position at which its output has no effect. Figure 14 shows this schematically.

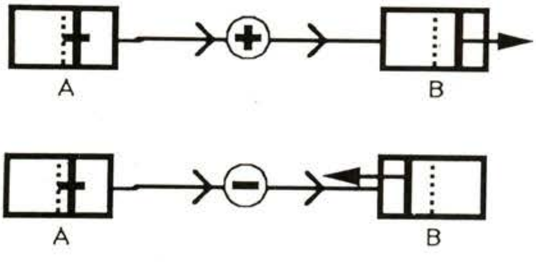

Fig. 14 Behaviour of linked variables in IQON

If variable 'A' is above 'neutral', a positive link from it to variable 'B' drives 'B' up progressively until it reaches the limit of its box. Similarly, a negative link to 'B' drives 'B' progressively down. Thus 'A' determines the rate of change of 'B'. Multiple inputs to a variable are simply averaged, taking account of sign, to determine the rate of change, though some inputs can be given greater weight than others. The response of each variable is made non-linear, through a 'squashing function' which restricts its values to the range minus one to plus one. A variable also has some (adjustable) internal damping. In fact, the behaviour is similar to that of some forms of artificial neuron (McClelland and Rumelhart 1987). One may of course also regard a variable as a (non-linear) integrator of its inputs.

These features mean that any system of inter-linked variables a user designs will have a smooth behaviour, with no tendency for variables to go to infinity or to produce large step function outputs, and that any system will have a unique stable condition from a given starting point.

Figures 15 and 16 show two examples of models created by pupils aged about 13 (Bliss and Ogborn 1992).

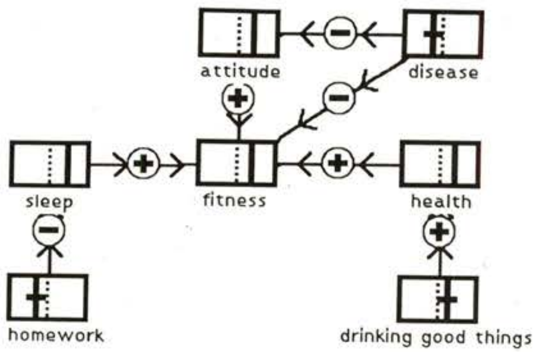

Fig. 15 Nancy's IQON model for keeping fit

Nancy (Figure 15) sees fitness depending both on general health and on whether one is getting plenty of sleep, and additionally on attitude. Jokingly, she says that if the school gives her a lot of work to do at home she gets less sleep. Health she sees as affected positively by sensible diet and negatively by disease, in both cases sliding a little away from quantitative variables towards events. Disease has a direct negative effect on fitness, and also an indirect effect via attitude. The point is not whether Nancy is right, but that she has produced a model which is discussable, and whose results when run may surprise her and lead her to think some more.

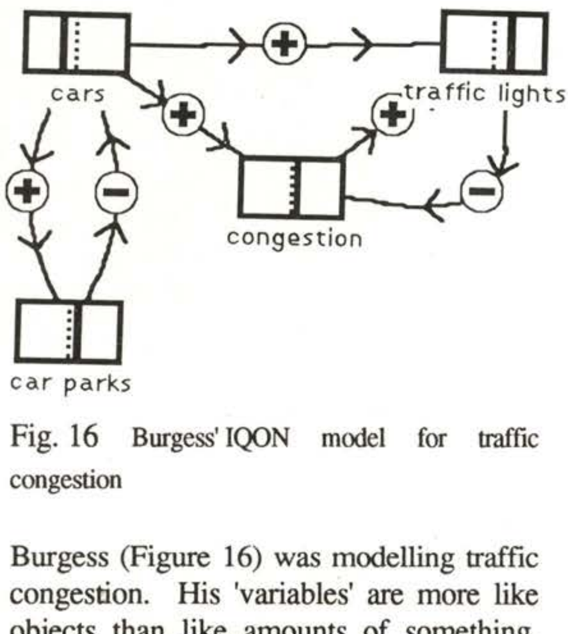

Fig. 16 Burgess' IQON model for traffic congestion

Burgess (Figure 16) was modelling traffic congestion. His 'variables' are more like objects than like amounts of something. Because of the feedbacks in the model, when it is run it can give surprising results. Increasing 'car parks' can at first decrease 'congestion' but, because of the loops between 'cars' and 'car parks' and between 'traffic lights' and 'congestion', the model is liable to oscillate. Again, what matters is that this is likely to lead the pupil to reconsider ideas.

Overall, the results of our studies with IQON (Bliss and Ogborn 1992) can be stated as follows:

- all pupils could make *some* model
- half or more made models with fairly sophisticated interconnections
- those who *made their own* models were more radical in criticising or reformulating them than were those who were given previously prepared models
- many had difficulty creating amount-like variables. The tendency was to create *objects* and *events*.
- some pupils could argue about feedback effects
- most pupils' work produced *discussable ideas,* capable of leading to progress in modelling.

In summary, we have a simple graphic modelling facility, for pupils to build such models out of just a few building bricks, and for them to be able to see some of the basic qualitative interactions at work, without yet having to consider exact functional relations between variables. The significant information is in the *qualitative pattern of relationship and change* amongst variables.

In Physics, one might *begin* with such qualitative models. Later, it would be time to see how well defined relationships in similar models can give more precise answers, in numerical simulations.

5 Modelling with objects and events

If one wants to make computational models with young pupils - say 8 to 12 years - then it would seem to be a good idea to model not variables but objects and events. We have been developing a modelling system for this purpose, called WorldMaker (Boohan, Ogborn and Wright, forthcoming).
A WorldMaker model of sharks preying on fish might look like Figure 17.

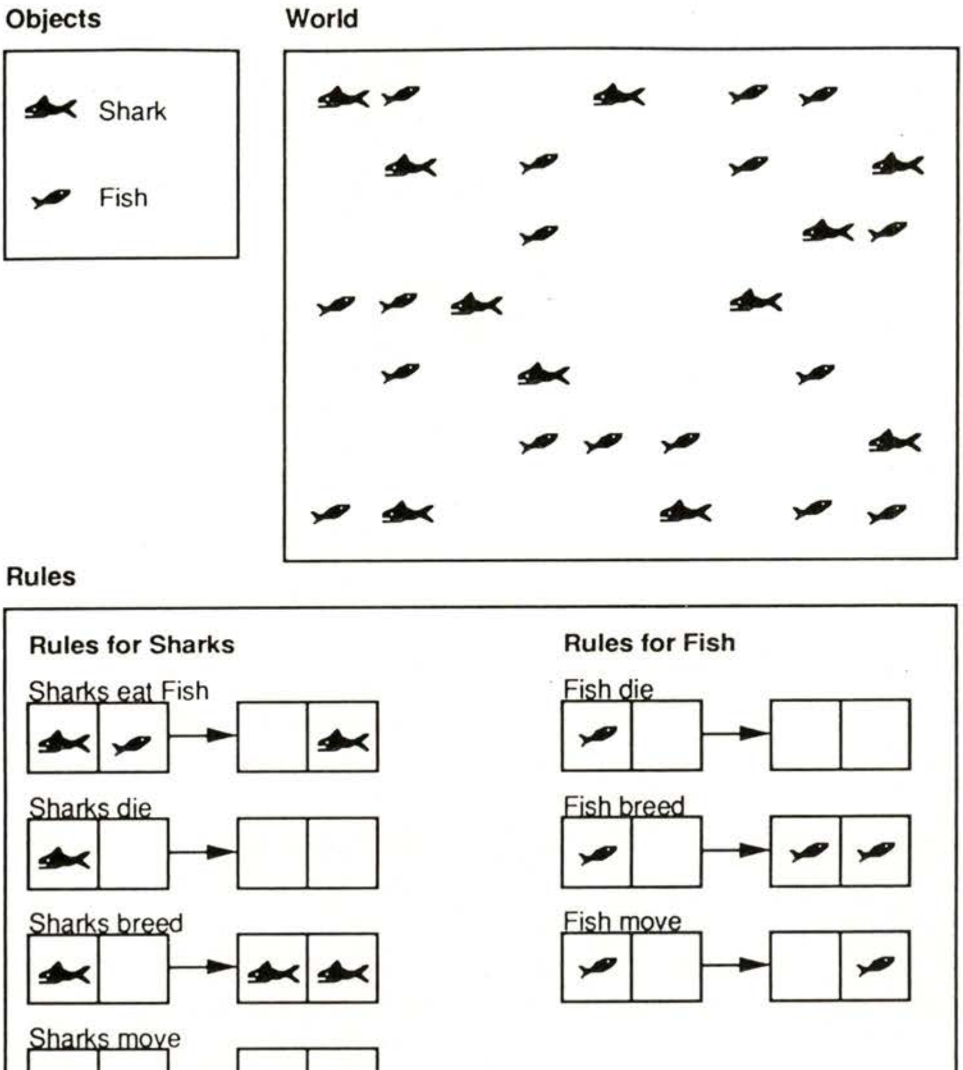

Fig. 17 Predator and prey in WorldMaker

A WorldMaker world consists of objects on a grid. Rules telling the objects what to do are defined graphically. Thus in Figure 17, the two kinds of object, sharks and fish, swim around the grid, being placed on it using drawing tools. Rules are specified by drawings, too. A shark next to an fish eats the fish. A shark on its own may die. A shark next to an empty space may breed or may move. The three rules for fish are similar to the last three rules for sharks. All rules have the form 'condition - effect'. Any rule can be set to 'fire' with a probability selected by a slider bar, so that for example relative breeding rates can be altered, or sharks can be made very long-lived. In this model, if sharks breed too fast, they can destroy the fish population and then themselves die out. As is well known, such predator-prey systems can oscillate.

The concept of WorldMaker derives from that of Von Neumann's cellular automaton (one of the best known instances being Conway's Game of Life), with the addition of moving objects each of which retains its identity, and of the possibility of random choices of allowed changes. A cell automaton consists of an array of cells, each of which has a small finite number of states. The state of a cell changes in relation to its own present state and those of its immediate neighbours. Thus the rules for evolution of the system are local rules, the same everywhere. A useful general account is given by Toffoli and Margolus (1987).

The system as a whole is not represented explicitly at all, but is visible to a person watching the model evolve, as some pattern of behaviour of the assembly of objects.

A simple model suitable for young pupils addresses the question why buses in town always seem to come in groups. Figure 18 shows the idea.

If buses stop to pick up people when they are there, the buses soon become clustered on the road around which they travel. WorldMaker allows directions of movement to be given to an object by the background it is on, making it simple to construct paths or tracks for objects. The example illustrates one of the several ways in which backgrounds and objects can interact, which include either changing the other into a different one. An example of such changes is a 'farmer' who moves around the grid 'planting crops' (i.e. changing bare earth to plants) and one or more 'pests' who move around destroying the crops. Another is shown in Figure 19, in which a creature moves purely at random, but moves frequently in the 'light' and rarely in the 'dark'. The result is that any initial distribution of creatures ends up with most of them in the 'dark' region.

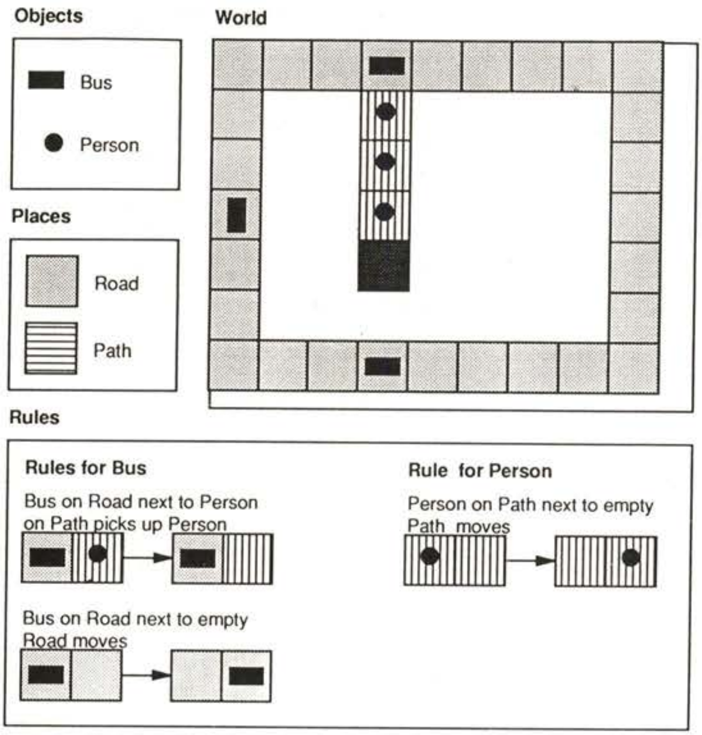

Fig. 18 WorldMaker for buses travelling in groups

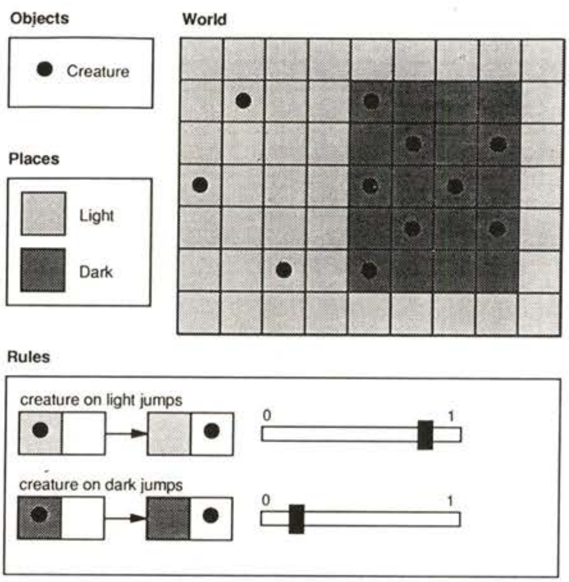

Fig. 19 WorldMaker model of preferential random distribution

An even simpler system, is able to illustrate molecular diffusion, as in Figure 20. The walls can be drawn anywhere one likes, and the initial distribution can be varied. The educational lesson here is important. A large scale, macroscopic appearance of systematic change can be generated by what is at the microscopic level random. Exactly the same rule will produce the outward diffusion of particles placed in a cluster at the centre of an otherwise empty screen.

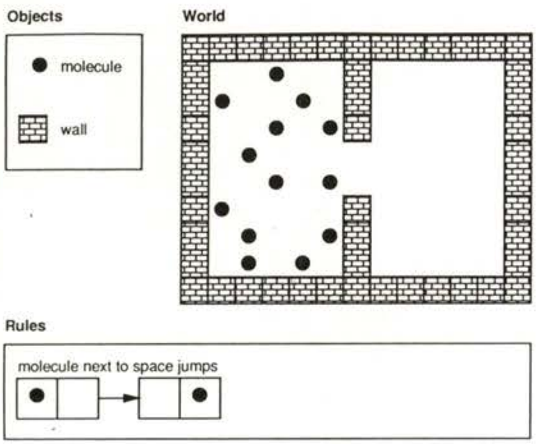

Fig. 20 WorldMaker model of molecular diffusion.

An adaptation of the model in Figure 20 leads to a model of diffusion limited aggregation. One just adds another object, a seed, which does not move, and the additional rule that a molecule alongside a seed is captured and turns into a seed. Figure 21 illustrates the kind of fractal structure which can result. It is not as impressive as the examples given by Professor Stauffer in his lecture (this issue) but students or teachers can make the model themselves. Other examples mentioned by Professor Stauffer can also be modelled, including cloud formation and the Ising model.

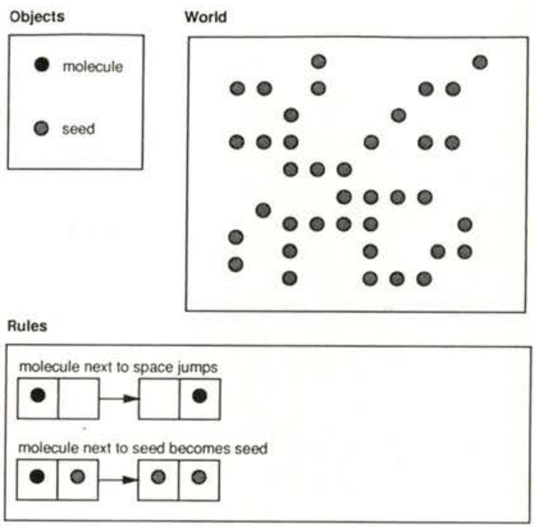

Fig. 21 WorldMaker model of diffusion limited aggregate

Let us mention some other models, simple and more advanced, which WorldMaker makes possible. One is radioactive decay, in which the rule us simply that an object representing a nucleus has a finite probability of changing to a stable nuclide. Such a model is` readily extended to a decay chain.

Marx (1984) gives the example of a forest fire, which belongs to the large class of percolation problems. A cell can be empty, or can contain a tree which is alive or is burnt. Trees are placed at random with a certain density over the screen, and one of them is 'set on fire' (figure 22). A tree burns if one or more of its neighbours burns. Will the fire travel all through the forest? It turns out that there is a critical density of trees for this to be likely. An equivalent problem is that of whether a mixture of conducting and insulating grains will be conducting, or of whether there are continuous percolation paths for oil through cracked rock strata. Marx (1984, 1987) gives many other interesting similar ideas.

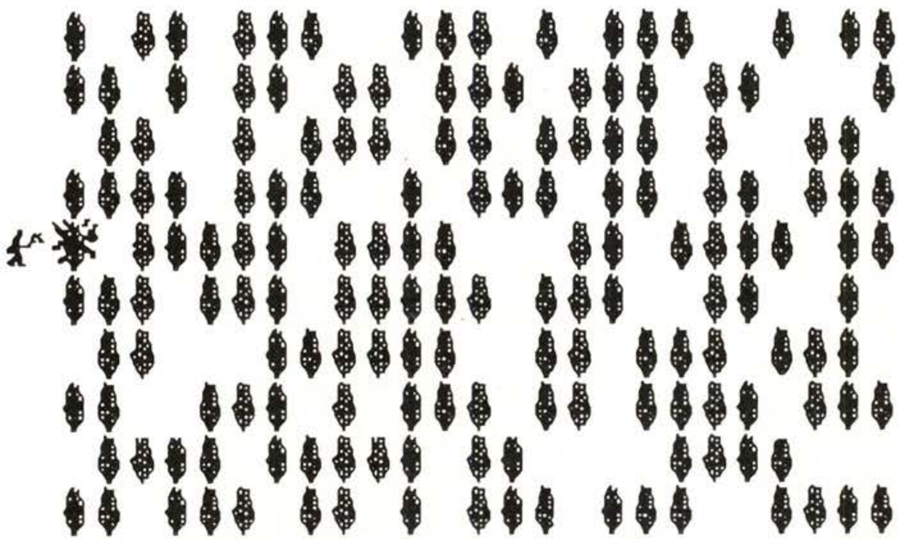

Fig. 22 Forest fire: one tree is set on fire - will all the forest burn?

Simple examples of chemical reactions can be modelled by having cells filled with two or more species of 'molecule'. Molecules may move to empty cells or may combine with others nearby to make product molecules, which themselves may react in the reverse direction.

In teaching thermodynamics, use can be made of models in which energy quanta move from particle to particle at random. Atkins (1984) describes a simple model in which cells have only two energy states, which offers an elegant introduction to temperature as understood statistically.

All these models have the great advantage that the *objects* one is talking about are directly represented on the computer screen. If the work concerns sharks eating fish, there are icons of sharks and fish to look at. If the problem is about molecules, one looks at an array of entities representing molecules, not at a display of variables such as temperature and pressure (though the system might in addition calculate these). The behaviour of the whole system is represented to the student by the visible pattern of behaviour of the objects, not as values of system variables. In general, the rules for the behaviour of entities are simple and intuitive, usually relating directly to their behaviour in the real world. Despite this simplicity, quite complex and analytically intractable systems can be studied.

6 Conclusions

I have in this paper suggested three things:

(a) that there is an important role in science teaching for quantitative system modelling;

(b) that there is scope for qualitative computational modelling of systems of variables.

(c) that use can be made of models which manipulate the objects in a system rather than the variables, and that cell automata provide a useful formalism for this concept;

Systems to provide for (a) already exist, and are in use in some schools, mainly in the upper age range. Those who cannot get or afford such a system, or who prefer an alternative already known to many pupils, can do a great deal with a spreadsheet program. The possibility is opened up of teaching Physics through modelling without having to wait until students know the calculus, and indeed of teaching the calculus in this way.

Suggestion (b) is more radical. We have built and tested a prototype, and can say that with it quite young pupils can produce interesting models. There are good psychological reasons for thinking that qualitative reasoning about variables is important, because of its pervasiveness in all human thought. The opportunity offers for teaching quite young students about systems of variables and effects of feedback, before they are ready to deal with quantitative formalised relations between variables.

Plenty of simulations which belong within the concept of (c) already exist, and are not difficult to program, though speed may be a problem. What I have suggested is the value of a generalized facility for building such models, and I have described one such system. Here we can see how the idea of modelling could be extended to pupils even in the Primary School.

I have tried to take a very broad view of what modelling with the computer might be, in the context of education. Thus let me finally try to put these thoughts in a more general perspective.

I will begin by noting that the normal order in which people come to appreciate the role of computational models, is far from ideal. The normal order is that first one is supposed to learn functional relations between quantities (Ohm's law, Newton's laws etc.), then some differential calculus, then integration, then numerical methods, and finally one is expected to see the unity in all this. This path is followed hardly any distance by most pupils, and the whole distance by almost none except the best doctoral students.

This leads me to propose in a sense to reverse the normal order. We should perhaps concentrate from the beginning on *form* , defined at first loosely and then more precisely. At present we leave form until last, if we ever reach it at all.

If it is true that children would find computational representations of *objects* easier to deal with than representations of *system variables* , then this suggests one kind of beginning with modelling in which the child tells the objects what to do, not the variables. *Form* is then represented by patterns of behaviour of collections of objects.

A second beginning, directed towards analysing systems into related variables, might be with modelling systems supporting qualitative reasoning, or patterns of cause and effect, involving variables. Here one has the possibility of looking at form as the typical kind of behaviour of systems with a given structure. The reason why oscillators oscillate is fundamentally the same. The reasons why stable systems are stable are often basically similar.

Thus, at this general level, I want to emphasize the very real importance, equally for young pupils and for the best experts, of qualitative reasoning about form. The young child can often guess how things may go, and can look at a model on the computer to see if it 'goes right' or not. The expert is an expert just by virtue of having passed beyond the essential stage of being able to do detailed calculations, to have reached the even more essential stage of knowing what kind of calculation to do, and what kind of result it will give.

To create a world, whether constituted of variables or of objects, and to watch it evolve is a remarkable experience. It can teach one what it means to have a model of reality, which is to say what it is to think. It can show both how good and how bad such models can be. And by becoming a game played for its own sake it can be a beginning of purely theoretical thinking about forms. The microcomputer brings something of this within the reach of most pupils and teachers.

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